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A Game Theoretic Framework for Power Allocation in Full-Duplex Wireless Networks

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ABSTRACT In this paper, we apply a game theoretic approach to power allocation in full-duplex orthogonal frequency division multiple access networks. Such networks exhibit a full-duplex base station, which allocates the same radio resources to a pair of half-duplex user equipment: one on the uplink, and one on the downlink. A theoretical doubling of the capacity is threatened by the interferences generated by full-duplex operation: self-interference and intra-cell co-channel interference. In our work, we propose three non-cooperative games aimed at tackling the intricate task of allocating power to scheduled pairs of user equipment, with objectives varying from improving user equipment performance to reducing power expenditure. The games have two sets of players: the users on the uplink, and the base station on the downlink. We use a special class of games, known as super-modular games, to draft different player utilities with different objectives. Via a set of exhaustive simulations, we assess the significance of power allocation in full-duplex wireless networks and determine its gains and limitations.

INDEX TERMS Game theory, full-duplex, scheduling, power allocation.

I. INTRODUCTION

As recently as 2010, full-duplex (FD) wireless technologies were still perceived to be impossible. The signal transmitted from a wireless device would overwhelm the signal being received on the same radio resource, thus masking it completely [1]. Nonetheless, the introduction of self-interference cancellation (SIC) techniques thereafter made full-duplex communications possible. The main challenge that faces extracting gains from full-duplex technologies lies within the interferences that the full-duplex network creates itself. In the full-duplex orthogonal frequency division multiple access (FD-OFDMA) network scenario we envision, a full-duplex base station (BS) will concurrently communicate with a pair of half-duplex (HD) user equipment (UE). This keeps the complexities of implementing full-duplex communications away from the UEs. As such, on a certain radio resource, one UE will be transmitting, its paired downlink UE will be receiving, and the BS being the full-duplex node would be simultaneously transmitting and receiving. This network would suffer from two major added types of interference: self-interference at the full-duplex node, and intra-cell co-channel interference at the downlink UEs.

The transmitted signal at a BS with full-duplex capabilities would overwhelm the signal being received from a UE on the uplink, which is typically multiple times weaker. This is known as self-interference, a phenomenon that degrades the performance of uplink UEs in full-duplex wireless networks. SIC techniques, a set of analog and digital signal interference cancellation technologies [2], make full-duplex communications feasible because of their effectiveness in battling this type of interference.

Another interference generated as a ramification of full-duplex communications is intra-cell co-channel interference. As pairs of uplink-downlink UEs share the same radio resources, the signal transmitted from an uplink UE will interfere on that being received by its downlink pair. This degrades the performance of the latter. As such, scheduling on the uplink and the downlink should be coordinated in order to pair between the uplink-downlink UEs which interfere the least upon each other.

In our work, we have two main objectives. First, to use game theory to propose a distributed approach to power allocation in full-duplex wireless networks. And second, to use power allocation to study, and improve, the gains

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achievable from full-duplex wireless communications. We start by introducing both greedy and fair scheduling algorithms for FD-OFDMA wireless networks. Afterwards, we put forward multiple game theory based proposals for power allocation. These games have different objectives: maximizing UE SINR, reducing network interferences, increasing energy efficiency and others. Because of the intricate relation between uplink and downlink transmissions in a full-duplex wireless network, it is impossible for one utility function to encompass all these objectives, and at the same time be impervious to different network scenarios. As a result, we put forward a game theoretic framework composed of three games which have different objectives and are suitable for different network scenarios. For each game, we prove that Nash equilibriums (NEs) exist, and that a repeated best response algorithm can be used to reach them. We perform thorough numerical simulations to assess the performance of our approaches in various interference conditions. The results show that power allocation has the ability to improve UE performance and save on expenditure, and that the pertinence of each our proposals depends on the scenario in question.

The remainder of this paper is structured as follows. Section II has the related works in the state-of-the-art and the contributions of this paper. Section III has the network model. The radio model used, the channel state information considered, as well as the UE traffic model are discussed in this section. In section IV, we present the scheduling and power allocation framework we used. Our greedy and fair scheduling proposals can be seen in section V, and an introduction into using non-cooperative game theory for power allocation is given in section VI. We propose three different games. The first, the greedy game, is presented in section VII. The second, the interference aware game, is presented is section VIII. And finally, the third, the energy efficient game, is put forward in section IX. Section X has the detailed simulation results. In it we go over the impact of scheduling and power allocation on UE throughput values and waiting delays, as well as the manner in which our proposed game theory based algorithms allocate power on the radio resources. Finally, this paper is concluded with section XI.

II. RELATED WORKS AND CONTRIBUTIONS

In this section, we look at the related works in the state-of-the-art. Research into full-duplex wireless communications can be classified into two major blocks. The first, constituted mainly of early works in the domain, focuses on verifying the possible gains of full-duplex wireless communications, whether through simulations or real-life modules. The second block thereafter focuses on devising scheduling and power allocation algorithms for full-duplex wireless networks.

The works in [3]–[6] revolve around assessing the gains of full-duplex wireless networks. Their authors study the limitations and obstacles of implementing full-duplex wireless communications. In one of the earliest works on in-band full-duplex for wireless networks, the authors in [3] surveyed a range of SIC techniques and touched on the main challenges facing full-duplex wireless networks. The authors in [4] proposed a full-duplex module with which they simulated two types of full-duplex networks: one where only the BS is full-duplex capable, and the other where both the UEs and the BS are full-duplex capable. Consequently, they assert the gains achievable from full-duplex communications. In [5], different scenarios and implementations of full-duplex networks are discussed. Mainly, four possible full-duplex wireless applications are presented. They include MIMO networks, cooperative networks, OFDMA networks, and Het-Nets. Again, the authors use resource management problems for the purpose of validating wireless full-duplex communications. With a more practical approach, the authors in [6] introduce a realistic model of a compact full-duplex receiver. With this model at hand, the authors demonstrate via numerical evaluations the capacity gains of full-duplex wireless networks and bring insights onto the impact of SIC on the performance of these networks. Finally, the articles in [7]–[9] refocus on the SIC techniques as the progress of cancellation technologies keeps growing the prospects of full-duplex wireless communications.

The second major block in the state-of-the-art tackles the tasks of scheduling and power allocation. The articles in [10]–[16] all focus on greedy scheduling algorithms based on Sum-Rate maximization coupled with optimal power allocation mechanisms. In some, heuristic algorithms were proposed to replace the mathematically intractable problem of jointly allocating radio resources and power. Probably the closest in the state-of-the-art to our objective in this paper, are the articles in [17] and [18]. The authors in [17] suggest that a game theoretic approach could be used for power allocation in full-duplex wireless networks. Their article surveys possible applications in relation to scheduling and power allocation in different full-duplex network scenarios. In [18], the authors use a game theoretic approach for resource allocation in full-duplex networks. While they implement a water filling algorithm for power control, their game theoretic approach focuses on greedy resource allocation with the purpose of sum-rate maximization.

In this paper, we propose a game theoretic framework for power allocation in FD-OFDMA wireless networks. We implement both fair and greedy resource allocation schemes in contrast to the majority of the state-of-the-art, and we seek via these scheduling algorithms to validate three different game proposals. In our work, we use non-cooperative game theory and show that each of our proposals converges to an NE via a repeated best response algorithm. Our system model uses a non-full buffer traffic model, unlike the vast majority of the state-of-the-art [10]–[18]. Dynamic traffic, like video streaming, would make upwards of 78 % of the mobile traffic worldwide by the year 2021 [19], signaling thus an importance in studying the implications of such traffic models. Furthermore, by using a non-cooperative game theoretic approach to power allocation, and by applying separate utilities for each set of players, our algorithms are significantly less complex than the mixed integer non-linear problems proposed in the articles mentioned above. This makes them easier to implement in real-life scenarios. We highlight our main contributions as follows:

(a) We propose multiple games for power allocation in full-duplex wireless networks. The games have different objectives and focus on maximizing different aspects of network performances.
(b) We implement queue-awareness into the scheduling and power allocation algorithms. This is avoided in the vast majority of the state-of-the-art on full-duplex wireless networks. We added buffer constraints to the optimization problems and counted for the fact that users might leave then rejoin the network.
(c) We propose a distributed approach to power allocation. This is also rather rare with respect to the state-of-the-art, where the majority of the papers implement centralized optimization algorithms.
(d) We studied the efficiency of power allocation in a full-duplex setting. While multiple papers introduce power allocation algorithms alongside their scheduling proposals, almost none study the efficiency or consequences of allocating power in full-duplex wireless networks.
(e) Via a set of exhaustive simulations, we compare and contrast between our proposals, highlighting their advantages, as well as their shortcomings.

III. NETWORK MODEL

A. RADIO MODEL

Our work is done with the assumption of a single-cell FD-OFDMA wireless network. This network includes a full-duplex BS and half-duplex UEs. The UEs in the network belong to one of two virtual sets: an uplink UE set $\mathcal{I}$ or a downlink UE set $\mathcal{D}$. The scheduler pairs between uplink and downlink UEs on every resource block (RB) k of the set of available RBs $\mathcal{K}$. The network model is shown in Fig. 1.

An OFDMA structure is used to operate the physical layer. The radio resources are divided into time-frequency RBs. In the time domain, an RB is comprised of an integer number of OFDM symbols. In the frequency domain, an RB has adjacent narrow-band subcarriers and experiences flat fading. The scheduling decision is taken for every Transmission Time Interval (TTI). At the start of every TTI, the scheduler has K RBs to allocate. The duration of a TTI is set to be less than the channel coherence time. As a result, UE radio conditions will vary from one RB to another but will remain constant over a select TTI. The modulation and coding scheme (MCS), that is assigned to a UE on a given RB, depends on its radio conditions. For performance evaluation, we consider LTE specifications, with an RB being composed of 12 subcarriers and 7 OFDM symbols [20].

We use an adapted formula to calculate the UE SINR values. This formula takes the co-channel interference between a UE pair and the self-interference cancellation performed by the BS into account. The SINR of uplink UE i observed on

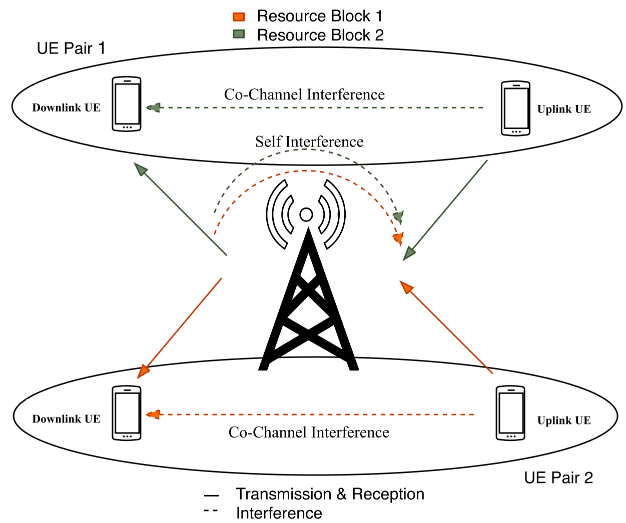

FIGURE 1. Network model and interferences.

RB k, whilst paired with downlink UE j, is expressed as:

$$S_j^u(i,k) = \frac{P_{ik}h_{ik}}{N_{0k} + \frac{P_{0k}}{SIC}}, \tag{1}$$

where on RB k, P_{ik} is the power emitted by UE i, h_{ik} is the channel gain between uplink UE i and the BS, and P_{0k} is the power emitted on the downlink by the BS on RB k. SIC denotes the self-interference cancellation performed by the BS, and thus $\frac{P_{0k}}{SIC}$ is the residual self-interference. Finally, N_{0k} is the noise power at the BS on RB k. Furthermore, the SINR observed by downlink UE j alloted RB k, and paired with uplink UE i, is expressed as:

$$S_i^d(j,k) = \frac{P_{0k}h_{0k}}{N_{jk} + P_{ik}h_{ij,k}}, \tag{2}$$

where h_{0k} is the channel gain between the downlink UE attributed RB k and the BS, and $h_{ij,k}$ is the channel gain between downlink UE j attributed RB k and interfering UE i, matched on that same RB. As such, $P_{ik}h_{ij,k}$ is the intra-cell co-channel interference affecting downlink UE j. Finally, N_{jk} is the noise power at downlink UE j allocated RB k.

B. CHANNEL STATE INFORMATION

In order to achieve reliable wireless communications, it is necessary for the scheduler to have knowledge on all of the channels in the network. Full-duplex communications pose an additional problem in this context. In full-duplex networks, information on the channels in between the pairs of UEs is required. In our work, we mathematically model the inter-UE channel as follows:

$$h_{ij,k} = G_t G_r L_p A_s A_f \tag{3}$$

G_t is the transmitter antenna gain. G_r is the receiver antenna gain. L_p represents the path loss. A_s represents the shadowing effects and A_f represents the fast fading effect. In this work,

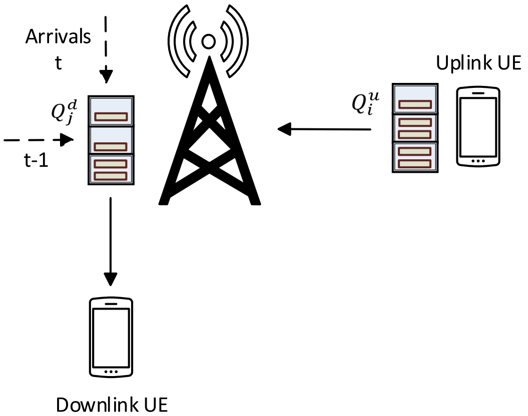

FIGURE 2. Traffic model: UE pair *i*-*j*.

the scheduler is assumed to have perfect channel state information. The impact of imperfect channel state information on full-duplex network performances has been investigated in our previous work in [21]. In it, we show that the lack of inter-UE channel state information could significantly decrease the full-duplex gains, and that partial knowledge of the channels would be enough to sustain these gains.

C. TRAFFIC MODEL

Our scheduling is queue-aware (Fig. 2). Each UE has a predefined throughput demand which determines the rate at which the UE will transmit or receive. A downlink UE has a queue at the BS, denoted Q_j^d, that it wants to receive. An uplink UE has a queue of bits it wants to transmit to the BS, denoted Q_i^u. UE queues are updated each TTI. They are filled according to a Poisson process with an average arrival rate λ equal to the throughput demand. Once the scheduling is done for a certain TTI, the number of bits each UE can transmit or receive is calculated, and the UE queues are deducted accordingly. The traffic is packeted into small units known as transport blocks. The modulation and coding scheme (MCS) that can be assigned to a UE is based on its SINR. Following the MCS used and the number of RBs allocated for a UE, its transport block size is determined for the TTI. Any bits remaining in a UE queue at the end of a TTI are carried on to the next one.

D. PERFORMANCE MODEL

The mapping between a UE's SINR and the number of bits it can transmit/receive is done following an MCS. Using LTE-like configurations, we set 15 channel quality indicator (CQI) values. The CQI values are used to identify the coding rates selected between 1/8 and 4/5, and the modulations chosen among 4-QAM, 16-QAM and 64-QAM. Figure 3 shows the mapping between the UE SINR values and the assigned CQI values.

Furthermore, Table 1 shows the relationship between the CQI level and the MCS schemes used. Based on the MCS used, the number of bits each UE can transmit or receive on the resources allocated to it is recorded. At the end of the simulation, the UE throughput is calculated as the number of bits the UE has transmitted divided by the simulation duration. Finally, the average UE waiting delay is calculated using Little's formula [22] as the average queue length divided by the packet arrival rate.

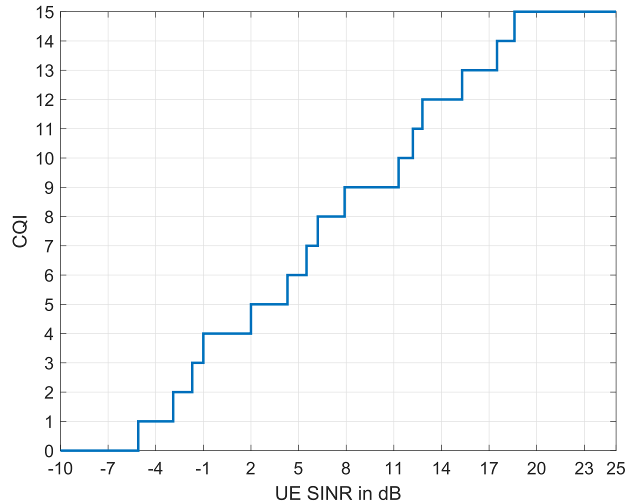

FIGURE 3. The CQI as a function of UE SINR.

TABLE 1. Modulation and coding scheme.

CQI	Modulation	Coding Rate
0	-	-
1	QPSK	1/8
2	QPSK	1/5
3	QPSK	1/4
4	QPSK	1/3
5	QPSK	1/2
6	QPSK	2/3
7	QPSK	3/4
8	QPSK	4/5
9	16-QAM	1/2
10	16-QAM	2/3
11	16-QAM	3/4
12	16-QAM	4/5
13	64-QAM	2/3
14	64-QAM	3/4
15	64-QAM	4/5

IV. PROBLEM SOLVING FRAMEWORK

In this paper, we propose three games for power allocation in FD-OFDMA wireless networks. First, the scheduler will allocate the resources to pairs of uplink-downlink UEs with the assumption of constant powers. Afterwards, our game theoretic proposals are used to calculate the transmit power on every allocated RB.

Given the UE SINR values and queues, and with constant power values, we compute the optimal resource allocation matrix z_{ijk}^*. The games for power allocation use this matrix as input and compute the power levels on the RBs. Afterwards, the scheduler recalculates the UE SINR values. The number of bits an uplink UE can transmit (T_{ijk}^u), or a downlink UE can receive (T_{ijk}^d), is calculated. The UE queues are then updated

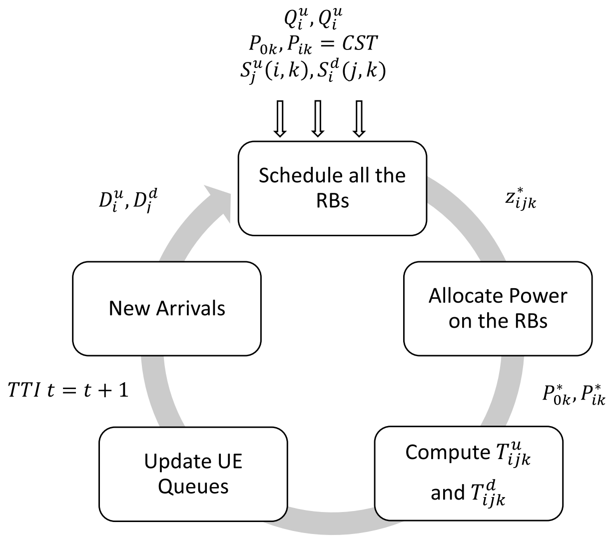

FIGURE 4. Problem solving framework for scheduling and power allocation.

depending on the resources each UE was allocated. At the beginning of the next TTI, and after the arrival of new bits, the UE demands D_i^u and D_j^d are updated.

V. SCHEDULING IN FD-OFDMA WIRELESS NETWORKS

As indicated in the framework, power allocation is done for scheduled pairs of UEs. In this section, we present both greedy and fair algorithms for scheduling in full-duplex wireless networks.

A. OPTIMAL FULL-DUPLEX MAX SINR

The objective of this algorithm is to allocate the RBs to the UE pairs with the highest sum of SINR values. The optimal formulation is illustrated below. (P_1^t):

$$\text{Maximize} \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{D}} z_{ijk}(S_j^u(i,k)+S_i^d(j,k)), \tag{4a}$$

$$\text{subject to} \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{D}} z_{ijk} \le 1, \quad \forall k\in\mathcal{K}, \tag{4b}$$

$$\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{D}} z_{ijk}T_{ijk}^u \le D_i^u, \quad \forall i\in\mathcal{I}, \tag{4c}$$

$$\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}} z_{ijk}T_{ijk}^d \le D_j^d, \quad \forall j\in\mathcal{D}, \tag{4d}$$

$$z_{ijk} \in \{0,1\}, \quad \forall i\in\mathcal{I},\ \forall j\in\mathcal{D}, \forall k\in\mathcal{K}. \tag{4e}$$

The resource assignment variable for the UE pairs denoted z_{ijk}, is defined $\forall\ k \in \mathcal{K}$, $\forall\ i \in \mathcal{U}$, $\forall\ j \in \mathcal{D}$. It is equal to one if uplink UE i is paired with downlink UE j on RB k, and zero otherwise. $S_j^u(i,k)$ and $S_i^d(j,k)$ are the SINR values for the uplink and downlink UEs of the pair, respectively.

T_{ijk}^u represents the number of bits uplink UE i can transmit on RB k while paired with downlink UE j. Similarly, T_{ijk}^d represents the number of bits UE j can receive on RB k in this scenario. These factors depend mainly on the radio conditions of the UEs. The demand of UE i, D_i^u, is the number of bits in its queue. Likewise, D_j^d is the demand of UE j.

Equation (4a) has the objective of our optimization problem which aims to select the pairs with the highest sum-SINR values. According to (4b), each RB should be allocated to either one or no pair. Equations (4c) and (4d) are the buffer constraints. They insure that a UE is allocated at maximum a number of resources sufficient to transmit or receive its queue and nothing more. Since our model is queue-aware, these constraints guarantee that the network resources are allocated efficiently.

B. OPTIMAL FULL-DUPLEX PROPORTIONAL FAIR

We aim to maximize the network's total throughput while at the same time ensuring a minimum level of fairness among the UEs. To this end, we propose a full-duplex Proportional Fair algorithm, which allocates RBs to the pairs of UEs with the highest sum of priorities. The priority of a UE is a function of its current and historic radio conditions, represented by the number of bits a UE can transmit, or receive, on the current RB vs. the number of bits it has already transmitted. The priority for a downlink UE j, paired with an uplink UE i on RB k, for example, is defined as:

$$\rho_i^d(j,k) = \frac{T_{ijk}^d}{T_j}, \tag{5}$$

where T_j is the number of bits UE j has received over a certain time window. The optimization problem for full-duplex Proportional Fair is presented in (P_2^t), where the objective function is to maximize the sum of priorities *i.e.*, select the pairs with the highest priorities. The constraints and assumptions from the previous problem remain the same.

(P_2^t):

$$\text{Maximize} \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{D}} z_{ijk}(\rho_j^u(i,k)+\rho_i^d(j,k)), \tag{6a}$$

$$\text{subject to} \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{D}} z_{ijk} \le 1, \quad \forall k\in\mathcal{K}, \tag{6b}$$

$$\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{D}} z_{ijk}T_{ijk}^u \le D_i^u, \quad \forall i\in\mathcal{I}, \tag{6c}$$

$$\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}} z_{ijk}T_{ijk}^d \le D_j^d, \quad \forall j\in\mathcal{D}, \tag{6d}$$

$$z_{ijk} \in \{0,1\}, \quad \forall i\in\mathcal{I},\ \forall j\in\mathcal{D},\ \forall k\in\mathcal{K}. \tag{6e}$$

C. HEURISTIC SCHEDULING

Our optimal formulations belong to the category of integer linear programming. While they can be efficiently solved using classical approaches such as branch and bound, they might still become mathematically intractable for a high number of variables. As such, we present a heuristic alternative (Algorithm 1) to the scheduling problems presented above. This algorithm allocates each RB to a pair of UEs in turn based on the sum of utilities F, where F is either the UE SINR or the UE priority. The matrix p_0^u has the initial transmit power of every uplink UE on every RB it was allocated.

p_0^d has the initial BS transmit power on all the RBs. Following the power allocation task, detailed later on in our game theoretic propositions, a queue update function "Update(x)" is called (Algorithm 2). This function deducts the number of transmitted/received bits from the corresponding UE queues. If a UE has emptied its queue, it is removed from the next scheduling iteration.

Algorithm 1 Heuristic Scheduling

Input: UE radio conditions, channel states, initial power settings p_0^u and p_0^d
For Each RB $k = 1, \ldots, K$
  Select a UE pair (i^*, j^*) such as
  $(i^*, j^*) = \underset{i \in \mathcal{I}, j \in \mathcal{D}}{\operatorname{argmax}}(F_j^u(i,k) + F_i^d(j,k))$
  Allocate RB k to pair (i^*, j^*)
End For
Allocate power following our proposals
For Every UE pair (i, j) Allocated an RB
  Update(i), Update(j)
End For

Algorithm 2 Queue Update Function

Update (x)
If $x \in \mathcal{I}$
  $Q_x^u \leftarrow Q_x^u - T_{xjk}^u$
  If $Q_x^u == 0$
    $\mathcal{I} \leftarrow \mathcal{I} - \{x\}$
  End If
End If
If $x \in \mathcal{D}$
  $Q_x^d \leftarrow Q_x^d - T_{ixk}^d$
  If $Q_x^d == 0$
    $\mathcal{D} \leftarrow \mathcal{D} - \{x\}$
  End If
End If

VI. NON-COOPERATIVE GAMES FOR POWER ALLOCATION

In our work, we use non-cooperative game theory to allocate power on the RBs. The latter models interactions between players competing for a common resource. It does not necessitate a central authority or any added signaling between the players of the game. Following the SINR formulas for uplink (1) and downlink (2) UEs, an increase in the power of an uplink UE will increase its SINR, but at the same time cause added interference on its paired downlink UE. Vice-versa, an increase in the transmit power at the BS, would increase the SINR of the receiving downlink UE, but cause added interference on the paired uplink UE. UEs on the uplink and the BS on the downlink, *i.e.*, the decision makers, are playing for contradicting objectives. Hence, non-cooperative game theory is well adapted to power allocation in full-duplex wireless networks.

A. GAME FORMULATION

We define a set of multi-player games $\mathcal{G}$ between the BS (coined player 0) and the $|\mathcal{I}|$ uplink UEs. In particular, on every allocated RB k, uplink UE i will compete with the BS. The formulation of such a non-cooperative game $\mathcal{G} = \langle M, S_0 \times \prod_i S_i, U \rangle$ can be described as follows:

- A finite set of players $M = (BS, UE\ \ i)$ are paired on the same RB k. In fact, on each allocated RB k, a two-players game is engaged between the BS and uplink UE i matched on RB k.
- The action of a given player is the amount of power allocated on RB k, the strategy chosen by the BS is then $\boldsymbol{P}_0 = (P_{01}, \ldots, P_{0|\mathcal{K}|})$ and the strategy chosen by any uplink UE i is $\boldsymbol{P}_i = (P_{i1}, \ldots, P_{i|\mathcal{K}|})$. A *strategy profile* $\boldsymbol{P} = (\boldsymbol{P}_0, \boldsymbol{P}_1, \ldots, \boldsymbol{P}_{|\mathcal{I}|})$ specifies the strategies of all players.
- For the BS, the space of pure strategies is S_0 given by what follows:
$$S_0 = \{\boldsymbol{P}_0 \in \mathbb{R}^{|\mathcal{K}|}, \text{ such as } \sum_{k \in \mathcal{K}} P_{0k} \leq p_0^{max} \text{ and } P_{0k} \geq p_0^{min}, \forall k \in \mathcal{K}\}$$
- For each uplink UE i, the space of pure strategies is S_i given by what follows:
$$S_i = \{\boldsymbol{P}_i \in \mathbb{R}^{|\mathcal{K}|}, \text{ such as } \sum_{k \in \mathcal{K}} P_{ik} \leq p_i^{max} \text{ and } P_{ik} \geq p_i^{min}, \forall k \in \mathcal{K}^i\}$$
$\mathcal{K}^i$ is the set of resources allocated to UE i and $S = S_0 \times S_1 \times \ldots \times S_{|\mathcal{I}|}$ is the set of all strategies.
- A set of utility functions $U = (U_0, U_{i \in \mathcal{I}})$ that quantify players' profit for a given strategy profile.

Note that an uplink player i will not transmit on an RB it was not allocated.

B. BEST RESPONSE

In non-cooperative game theory, a rational solution is one where all competing players adhere to an NE [23]. An NE is a profile of strategies in which no player will take advantage of the others by deviating its strategy unilaterally. Thus, the primary challenge in non-cooperative game theory is to propose algorithms capable of reaching such an equilibrium. The simplest of these algorithms are the repeated best response dynamics. Following these dynamics, each player selects the best, and locally optimal, response to other players' strategies, until the algorithm converges.

VII. THE GREEDY GAME

We define the greedy game $\mathcal{G}^g$. The objective of this game is to maximize UE SINR values. Both UEs on the uplink and the BS on the downlink will individually aim to increase the UE SINR values. Let $j(i, k)$ be a reference to downlink UE j paired with uplink UE i on RB k as a result of scheduling. For simplicity, in the remainder of this paper we use $j = j(i, k)$.

The utility of any uplink UE $i \in \mathcal{I}$ only encompasses its own SINR. It is formulated as follows:

$$U_i^g = \sum_{k \in \mathcal{K}^i} \log(\frac{P_{ik}h_{ik}}{N_{0k} + \frac{P_{0k}}{SIC}}), \tag{7}$$

where $\mathcal{K}^i$ is the set of RBs scheduled to UE i. The utility of the BS encompasses the SINR of both uplink and downlink UEs:

$$U_0^g = \sum_{k \in \mathcal{K}} (\log(\frac{P_{ik}h_{ik}}{N_{0k} + \frac{P_{0k}}{SIC}}) + \log(\frac{P_{0k}h_{0k}}{N_{jk} + P_{ik}h_{ij,k}})) \tag{8}$$

For any uplink UE i, U_i^g is concave in P_{ik} (logarithmic function) and continuous in $P_{0k}, \forall k \in \mathcal{K}$. For the BS, U_0^g is concave in P_{0k} since

$$\frac{\partial^2 U_0^g}{\partial P_{0k}^2} = -\frac{SIC \cdot N_{0k} \times (SIC \cdot N_{0k} + 2P_{0k})}{P_{0k}^2 \times (N_{0k} \cdot SIC + P_{0k})^2} < 0, \tag{9}$$

and continuous in P_{ik}, ∀ k ∈ $\mathcal{K}$. Hence, as all strategy spaces are compact, an NE exists for this game.

A. COMPUTING A NASH EQUILIBRIUM

As the utility functions are strictly concave, the NE is the solution of the following two optimization problems:

$$\max_{\boldsymbol{P}_\gamma} U_\gamma^g(\boldsymbol{P}_\gamma, \boldsymbol{P}_{-\gamma}), \tag{10a}$$

$$\text{subject to} \sum_{k \in \mathcal{K}} P_{\gamma k} \leq p_\gamma^{max}, \tag{10b}$$

$$P_{\gamma k} \geq p_\gamma^{min}, \quad \forall k \in \mathcal{K}. \tag{10c}$$

where p_γ^{max} (resp. p_γ^{min}) is the maximal (resp. minimal) power limit on the uplink for $\gamma \in \mathcal{I}$ and on the downlink for $\gamma = 0$.

As the optimization problems in (10) are nonlinear and convex, the Karush-Kuhn-Tucker (KKT) conditions are sufficient to determine the optimal case (*i.e.* the NE) [24]. The KKT conditions associated with $P_{ik}, \forall k \in \mathcal{K}^i$ for uplink UE $i \in \mathcal{I}$ give what follows:

$$\frac{1}{P_{ik}^*} - \mu = 0, \quad \forall k \in \mathcal{K}^i, \tag{11a}$$

$$\mu \times (p_i^{max} - \sum_{k \in \mathcal{K}^i} P_{ik}^*) = 0, \tag{11b}$$

$$P_{ik}^* \geq p_i^{min}, \quad \forall k \in \mathcal{K}^i, \tag{11c}$$

$$\sum_{k \in \mathcal{K}^i} P_{ik}^* \leq p_i^{max}, \tag{11d}$$

$$\mu \geq 0, \tag{11e}$$

where μ is the Lagrange multiplier associated with the constraint (10b). We deduce from (11a) that μ cannot be null and hence all P_{ik}^* are equal. Furthermore, according to (11b), $\sum_{k \in \mathcal{K}^i} P_{ik}^* = p_i^{max}$ and finally $P_{ik}^* = max(p_i^{min}, \frac{p_i^{max}}{\mathcal{K}^i})$ if RB k is allocated to UE i and 0 otherwise.

The KKT conditions associated with $P_{0k}, \forall k \in \mathcal{K}$ for the BS give what follows:

$$\frac{1}{P_{0k}^*} - \frac{1}{N_{0k} \cdot SIC + P_{0k}^*} - \mu + \alpha_k = 0, \quad \forall k \in \mathcal{K}, \tag{12a}$$

$$\mu \times (p_0^{max} - \sum_{k \in \mathcal{K}} P_{0k}^*) = 0 \tag{12b}$$

$$\alpha_k \times (p_0^{min} - P_{0k}^*) = 0, \quad \forall k \in \mathcal{K} \tag{12c}$$

$$P_{0k}^* \geq p_0^{min}, \quad \forall k \in \mathcal{K}, \tag{12d}$$

$$\sum_{k \in \mathcal{K}} P_{0k}^* \leq p_0^{max}, \tag{12e}$$

$$\alpha_k \geq 0, \quad \forall k \in \mathcal{K}, \tag{12f}$$

$$\mu \geq 0, \tag{12g}$$

where μ and $\alpha_k, \forall k \in \mathcal{K}$ are the Lagrange multiplier associated with the constraints (10b) and (10c) respectively. We deduce from (12a) that μ cannot be null and therefore the BS will allocate greedily all available power as

$$\sum_{k \in \mathcal{K}} P_{0k}^* = p_0^{max} \tag{13}$$

Furthermore, we need to distinguish between two cases where $\alpha_k, \forall k \in \mathcal{K}$ are either null or not.

1) If $\alpha_k \neq 0$, $P_{0k}^* = p_0^{min}, \forall k \in \mathcal{K}$ according to (12c). Further, in conjunction with (13), maximal and minimal thresholds should verify $\frac{p_0^{max}}{K} = p_0^{min}$.
2) If $\alpha_k = 0, \forall k \in \mathcal{K}$, we need to solve the following second order equation: $\frac{a_k}{P_{0k}^* \times (a_k + P_{0k}^*)} = \mu, \forall k \in \mathcal{K}$, where $a_k = N_{0k} \cdot SIC$ which gives a single realistic solution (positive power value) $P_{0k}^* = \frac{a_k \cdot (\sqrt{1 + \frac{4}{a_k \mu}} - 1)}{2}$. As the value of power levels is still dependent on the Lagrangian variable μ (the a_k are constant and known), we have recourse to (13) to obtain it and compute the numerical values of $P_{0k}^*, \forall k \in \mathcal{K}$ accordingly.

B. BEST RESPONSE ALGORITHM

A best response algorithm, illustrated in Algorithm 3, is used in this case to reach an NE. After the resources are allocated, the uplink UEs and the BS take turn maximizing their utilities until the power values on the RBs no longer change. The resulting power allocation scheme is used in performance assessments. p_n^u and p_n^d are matrices containing all the power values on the RBs on the uplink and the downlink, respectively during iteration n.

VIII. THE INTERFERENCE AWARE GAME

We define the interference aware collaborative game $\mathcal{G}^c$. Our objective in this game is for power allocation to be interference aware. Since the game is non-cooperative, it is necessary that each player is aware of the interferences they generate. If these interferences are not accounted for in the utilities, each player will seek to maximize its own gains independently, and consequently, increase its transmit power. This would generate maximum interference in the network and could inadvertently degrade UE performance. The utility of every uplink UE i is thereafter written as:

$$U_i^c = \sum_{k \in \mathcal{K}^i} \log(\frac{P_{ik}h_{ik}}{N_{0k} + \frac{P_{0k}}{SIC} + P_{ik}h_{ij,k}}). \tag{14}$$

Algorithm 3 Scheduling and Power Allocation Algorithm for the Greedy Game

Requires: Maximum tolerance $\epsilon \geq 0$.
Input: UE radio conditions, channel states, initial power settings p_0^u and p_0^d
For TTI $t = 1, \ldots, T$
  Step 1: Scheduling
  RBs are allocated following (P_1^t) or (P_2^t)
  Step 2: Power Allocation
  Repeat:
    Solve (10) on the uplink $\forall\, i \in \mathcal{I}$
    Update p_n^u
    Solve (10) on the downlink for the BS
    Update p_n^d
    $\delta^d = \|p_n^d - p_{n-1}^d\|$, $\delta^u = \|p_n^u - p_{n-1}^u\|$
    $n \leftarrow n + 1$
  Until $\delta^d \leq \epsilon$ and $\delta^u \leq \epsilon$
  Step 3: Update UE Queues
End For

As for the BS:

$$U_0^c = \sum_{k \in \mathcal{K}} \log\left(\frac{P_{0k} h_{0k}}{N_{jk} + P_{ik} h_{ij,k} + \frac{P_{0k}}{SIC}}\right). \tag{15}$$

The SINR for the UEs, on the uplink and on the downlink, are thus inherently included. Additionally, the co-channel interference, which degrades the performance of downlink UEs, is now also affecting the utility of uplink UEs. The self-interference, which degrades the performance of uplink UEs, is now also affecting the utilities relating to downlink UEs. As such, we can seek to improve UE performance, while at the same time account for the resulting interferences. Via our simulations, we show that our proposed utilities converge to an efficient NE which improves UE performance.

A. A SUPER-MODULAR GAME

It is not always guaranteed that a best response algorithm converges. This game is in fact part of a special class of games known as super-modular. In such games, a best response algorithm permits achieving NEs. In what follows, we give a definition of super-modular games and prove that our power allocation game belongs to this class. According to [25], any game $\mathcal{G}$ is super-modular if for any player $\gamma \in M$:

1) The strategy space S_γ is a compact sub-lattice of $\mathbb{R}^{|\mathcal{K}|}$;
2) The objective function is super-modular, that is $\frac{\partial^2 U_0}{\partial P_0 \partial P_i} \geq 0$ and $\frac{\partial^2 U_i}{\partial P_i \partial P_0} \geq 0\ \forall i \in \mathcal{I}, \forall \boldsymbol{P} \in S$, and $\forall k \in \mathcal{K}$.

In [25], [26], proof is given for the following two results in a super-modular game:

- If each player γ initially uses either its lowest or largest policy in S_γ, then a best response algorithm will converge monotonically towards an NE.
- Starting with a feasible policy, then the sequence of best responses will converge to an NE: in a super-modular game, it monotonically increases in all components in the case of maximization.

Proposition 1: Game $\mathcal{G}^c \left\langle M, S_0 \times \prod_i S_i, U^c \right\rangle$ is a super-modular game.

Proof: To prove the super-modularity of our game, we need to verify the conditions mentioned above. First, the strategy space S_γ is evidently a compact convex set of $\mathbb{R}^{|\mathcal{K}|}$. Hence, it suffices to verify the super-modularity of the objective function U_γ^c of any player γ as there are no constraint policies for $\mathcal{G}^c$. For any uplink UE i, we have:

$$\frac{\partial^2 U_i^c}{\partial P_{ik} \partial P_{0k}} = \frac{\frac{h_{ij,k}}{SIC}}{(N_{0k} + \frac{P_{0k}}{SIC} + P_{ik} h_{ij,k})^2} \geq 0, \quad \forall i \in \mathcal{I},\ \forall k \in \mathcal{K}. \tag{16}$$

And for the BS, we have what follows:

$$\frac{\partial^2 U_0^c}{\partial P_{0k} \partial P_{ik}} = \frac{\frac{h_{ij,k}}{SIC}}{(N_{jk} + \frac{P_{0k}}{SIC} + P_{ik} h_{ij,k})^2} \geq 0, \quad \forall i \in \mathcal{I},\ \forall k \in \mathcal{K}. \tag{17}$$

■

B. COMPUTING A NASH EQUILIBRIUM

As we proved that our game is super-modular game, we implement a best response algorithm to reach its pure NE. At the convergence of the best response algorithm, an NE is the solution of the following optimization problems:

$$\max_{\boldsymbol{P}_\gamma}\ U_\gamma^c(\boldsymbol{P}_\gamma, \boldsymbol{P}_{-\gamma}) \tag{18a}$$

$$\text{subject to} \sum_{k \in \mathcal{K}} P_{\gamma k} \leq p_\gamma^{max}, \tag{18b}$$

$$P_{\gamma k} \geq p_\gamma^{min}, \quad \forall k \in \mathcal{K}. \tag{18c}$$

where p_γ^{max} (resp. p_γ^{min}) is the maximal (resp. minimal) power limit on the uplink for $\gamma \in \mathcal{I}$ and on the downlink for $\gamma = 0$. As the optimization problems in (18) are convex, the Karush-Kuhn-Tucker (KKT) conditions enable determining a global optimal (*i.e.*, the NE at convergence) [24]. The KKT conditions associated with $P_{\gamma k}, \forall k \in \mathcal{K}$ give what follows:

$$\frac{1}{P_{\gamma k}^*} - \frac{1}{b_{\gamma k} + P_{\gamma k}^*} = \lambda_\gamma, \quad \forall k \in \mathcal{K}, \tag{19a}$$

$$\lambda_\gamma \times (p_\gamma^{max} - \sum_{k \in \mathcal{K}} P_{\gamma k}^*) = 0, \tag{19b}$$

$$P_{\gamma k}^* \geq p_\gamma^{min}, \quad \forall k \in \mathcal{K}, \tag{19c}$$

$$\sum_{k \in \mathcal{K}} P_{\gamma k}^* \leq p_\gamma^{max}, \tag{19d}$$

$$\lambda_\gamma \geq 0, \tag{19e}$$

where λ_γ is the KKT multiplier associated with the constraint (18b), and

$$b_{\gamma k} = \begin{cases} \dfrac{N_{0k} + \frac{P_{0k}^*}{SIC}}{h_{ij,k}}, & \gamma = i \in \mathcal{I} \\ SIC \times (N_{jk} + P_{ik}^* h_{ij,k}), & \gamma = 0 \end{cases} \tag{20}$$

Algorithm 4 Scheduling and Power Allocation Algorithm for the Interference-Aware Game

Requires: Maximum tolerance $\epsilon \geq 0$.
Input: UE radio conditions, channel states, initial power settings p_0^u and p_0^d
For TTI $t = 1, \ldots, T$
  Step 1: Scheduling
  RBs are allocated following (P_1^t) or (P_2^t)
  Step 2: Power Allocation
  Repeat:
    Solve (18) on the uplink $\forall\, i \in \mathcal{I}$
    Update p_n^u
    Solve (18) on the downlink for the BS
    Update p_n^d
    $\delta^d = \|p_n^d - p_{n-1}^d\|$, $\delta^u = \|p_n^u - p_{n-1}^u\|$
    $n \leftarrow n+1$
  Until $\delta^d \leq \epsilon$ and $\delta^u \leq \epsilon$
  Step 3: Update UE Queues
End For

We deduce from (19a) that λ_γ cannot be null. As such, all $P^*_{\gamma k}$ are the solution of a second order equation that gives $P^*_{\gamma k} = \frac{b_{\gamma k}\cdot(\sqrt{1+\frac{4}{b_{\gamma k}\lambda_\gamma}}-1)}{2}$, where λ_γ can be computed numerically owing to $\sum_{k\in\mathcal{K}} P^*_{\gamma k} = p_\gamma^{max}$. Finally, in respect with constraint (19c), we have what follows for the BS:

$$P^*_{0k} = \max(p_0^{min}, \frac{SIC \times (N_{jk} + P^*_{ik}h_{ij,k})}{2} \cdot(\sqrt{1 + \frac{4}{SIC \cdot (N_{jk} + P^*_{ik}h_{ij,k})\lambda_0}} - 1)), \tag{21}$$

and for any uplink UE i:

$$P^*_{ik} = \max(p_i^{min}, \frac{N_{0k} + \frac{P^*_{0k}}{SIC}}{2h_{ij,k}} \cdot (\sqrt{1 + \frac{4}{\frac{N_{0k}+\frac{P^*_{0k}}{SIC}}{h_{ij,k}}\lambda_i}} - 1)). \tag{22}$$

C. BEST RESPONSE ALGORITHM

Algorithm 4 has the scheduling and power allocation algorithm. Typically, the algorithm will reach an NE for the power allocation step in 3 to 4 iterations.

IX. THE ENERGY EFFICIENT GAME

Our objective in this game is to avoid power wastage as much as feasible. Our energy efficiency objective represents a benefit-to-cost ratio, where the benefit is represented by the SINR of the UEs and the cost by the interferences generated by them. To this end, any player γ weights its SINR by the interference it creates. Accordingly, for the energy efficient game $\mathcal{G}^e$, the utility of every uplink UE i becomes:

$$U_i^e = \frac{\sum_{k\in\mathcal{K}^i} \log \frac{P_{ik}h_{ik}}{N_{0k}+\frac{P_{0k}}{SIC}}}{\sum_{k\in\mathcal{K}^i} P_{ik}h_{ij,k}} \tag{23}$$

As for the BS:

$$U_0^e = \frac{\sum_{k\in\mathcal{K}} \log \frac{P_{0k}h_{0k}}{N_{jk}+P_{ik}h_{ij,k}}}{\sum_{k\in\mathcal{K}} \frac{P_{0k}}{SIC}} \tag{24}$$

Proposition 2: Game $\mathcal{G}^e \left\langle M, S_0 \times \prod_i S_i, U^e \right\rangle$ is a super-modular game.

Proof: To prove the super-modularity of the game, we need to verify the conditions discussed in the previous section. First, the strategy space S_γ is also a compact convex set of $\mathbb{R}^{|\mathcal{K}|}$. Hence, it suffices to verify the super-modularity of the objective function U_γ^e of any player γ as there are no constraint policies for $\mathcal{G}^e$. For any uplink UE i, we have:

$$\frac{\partial^2 U_i^e}{\partial P_{ik}\partial P_{0k}} = \frac{h_{ij,k}}{(\sum_{k\in\mathcal{K}^i} P_{ik}h_{ij,k})^2(SIC \cdot N_{0k} + P_{0k})} \geq 0, \quad \forall i \in \mathcal{I}, \forall k \in \mathcal{K}.$$

And for the BS, we have what follows:

$$\frac{\partial^2 U_0^e}{\partial P_{0k}\partial P_{ik}} = \frac{h_{ij,k}}{SIC \cdot (N_{jk} + P_{ik}h_{ij,k})(\sum_{k\in\mathcal{K}} \frac{P_{0k}}{SIC})^2} \geq 0, \quad \forall i \in \mathcal{I}, \forall k \in \mathcal{K}.$$

■

A. COMPUTING THE NE

As we proved that our game is super-modular, we implement a best response algorithm to reach its pure NEs. An NE is the solution of the following optimization problem:

$$\max_{\boldsymbol{P}_\gamma} U_\gamma^e(\boldsymbol{P}_\gamma, \boldsymbol{P}_{-\gamma}) \tag{25a}$$

$$\text{subject to} \sum_{k\in\mathcal{K}} P_{\gamma k} \leq p_\gamma^{max}, \tag{25b}$$

$$P_{\gamma k} \geq p_\gamma^{min}, \ \forall k \in \mathcal{K}. \tag{25c}$$

where p_γ^{max} (resp. p_γ^{min}) is the maximal (resp. minimal) power limit on the uplink for $\gamma \in \mathcal{I}$ and on the downlink for $\gamma = 0$.

B. DINKELBACH APPROACH

The problem presented above is non-convex and as such cannot be solved in a straightforward manner. Nonetheless, it is a fractional problem and an optimal solution could be obtained by iteratively solving the parametrized convex problem according to the Dinkelbach method. For each uplink UE, the objective is rewritten as follows:

$$\max_{\boldsymbol{P}_{ik}} F(\theta_i) = \sum_{k\in\mathcal{K}^i} \log \frac{P_{ik}h_{ik}}{N_{0k} + \frac{P_{0k}}{SIC}} - \theta_i \sum_{k\in\mathcal{K}^i} P_{ik}h_{ij,k}, \tag{26a}$$

$$\text{subject to} \sum_{k\in\mathcal{K}^i} P_{ik} \leq p_i^{max}, \tag{26b}$$

$$P_{ik} \geq p_i^{min}, \quad \forall k \in \mathcal{K}^i. \tag{26c}$$

And for the BS on the downlink, it can be rewritten as follows:

$$\max_{\boldsymbol{P}_{0k}} F(\theta_0) = \sum_{k\in\mathcal{K}} \log \frac{P_{0k}h_{0k}}{N_{jk}+P_{ik}h_{ij,k}} - \theta_0 \sum_{k\in\mathcal{K}} \frac{P_{0k}}{SIC}, \tag{27a}$$

$$\text{subject to} \sum_{k\in\mathcal{K}} P_{0k} \leq p_0^{max}, \quad (27b)$$

$$P_{0k} \geq p_0^{min}, \quad \forall k \in \mathcal{K}. \quad (27c)$$

The values of θ_0 and θ_i can be calculated by iteration. The problems in (26) and (27) are first solved for large values of θ_i and θ_0, respectively. Afterwards, θ_γ is calculated assuming that the objective functions found in (26a) and (27a) are equal to zero. The problems in (26) and (27) are solved and the steps are repeated until the values of θ_i and θ_0 satisfy the conditions $F(\theta_i) = 0$, and $F(\theta_0) = 0$, respectively. This process is illustrated in Algorithm 5.

Algorithm 5 Calculating θ_γ

Set: $\theta_\gamma = \mathcal{M}$ a sufficiently large value
    Repeat:
        Solve the problem in (26) for $\gamma = i$
        Solve the problem in (27) for $\gamma = 0$
        Compute θ_γ such as $F(\theta_\gamma) = 0$
    Until $F(\theta_\gamma) = 0$

Algorithm 6 Scheduling and Power Allocation Algorithm for the Energy Efficient Game

Requires: Maximum tolerance $\epsilon \geq 0$.
Input: UE radio conditions, channel states, initial power settings p_0^u and p_0^d
For TTI $t = 1, \ldots, T$
    Step 1: Scheduling
    RBs are allocated following (P_1^t) or (P_2^t)
    Step 2: Power Allocation
    Repeat:
        Using Algorithm 5 get the value of $\theta_i \; \forall \; i$
        Solve (26) on the uplink $\forall \; i \in \mathcal{I}$
        Update p_n^u
        Using Algorithm 5 get the value of θ_0
        Solve (27) on the downlink for the BS
        Update p_n^d
        $\delta^d = \|p_n^d - p_{n-1}^d\|$, $\delta^u = \|p_n^u - p_{n-1}^u\|$
        $n \leftarrow n+1$
    Until $\delta^d \leq \epsilon$ and $\delta^u \leq \epsilon$
    Step 3: Update UE Queues
End For

C. BEST RESPONSE

Similar to the game before, a best response algorithm is used to reach an NE. Uplink players and the BS take turns solving the Dinkelbach problem for their locally optimal powers, until convergence is reached wherein the power levels on the RBs no longer change. This is illustrated in Algorithm 6.

X. SIMULATIONS AND RESULTS

A. SIMULATION PARAMETERS

We seek via our different simulation scenarios to address gains attributed to our game theoretic proposal. The simulation parameters we used are presented in Table 2.

The channel gain includes the shadowing, the path loss, and the fast fading effects. The path loss is determined using the extended Hata path loss model [27]. The fast fading is modeled by an exponential random variable A_f with unit parameter. The shadowing is modeled using a log-normal random variable $A_s = 10^{(\frac{\xi}{10})}$, where ξ is a normal distributed random variable with zero mean and standard deviation equal to 10. This model, which is used for urban zones, takes into account the effects of reflection, diffraction and scattering caused by city structures. Note that the cell radius is chosen to be 120 m, unless specified otherwise.

TABLE 2. Simulation parameters.

Parameter	Value
Cell Specifications	Single-Cell, 120-1000 m Radius
Number of RBs	60
BS Transmit Power	24 dBm
Maximum UE Transmit Power	24 dBm
SIC Value	10^{11}
Number of UEs	20 UEs: 10 downlink, 10 uplink
UE Distribution	Uniform
Demand Throughput	4 Mbps
Fast Fading	Rayleigh. σ=1
Shadowing	Normal law. μ=0 dB σ^2=10 dB
Path Loss Model	Extended Hata Path Loss Model

B. EFFECT OF THE SCHEDULING OBJECTIVE ON UE THROUGHPUT

In this section, we study the effect of the scheduling objective on UE performance. We simulate both our scheduling algorithms full-duplex Max SINR and full-duplex Proportional Fair, alongside the power allocation proposal presented in the greedy game in section VII. We also simulate the full-duplex Max Sum-Rate (SR) scheduling algorithm given by the authors in [10] alongside the same power allocation proposal. Finally, for the sake of comparison with current half-duplex wireless transmissions, both traditional half-duplex Max SINR and half-duplex Proportional Fair algorithms are simulated. Maximum power allocation is assumed for the latter two. The cell radius is set at 120 m. Figure 5 has box plots with the resulting UE throughput values.

For both the full-duplex and half-duplex algorithms, the effect of the scheduling objectives on the resulting UE throughput values is evident. Max SINR scheduling results in an uneven distribution of the RBs, which can be seen in the large box sizes. On the other hand, Proportional Fair scheduling distributes the network's resources more evenly. This can be seen in the relatively smaller boxes.

Comparing half-duplex to full-duplex scheduling, it can be noted that the latter produces almost double the throughput values. Half-duplex Proportional Fair scheduling results in a median throughput value close to 0.8 Mbps compared to about 1.9 Mbps for its full-duplex counterpart. Half-duplex

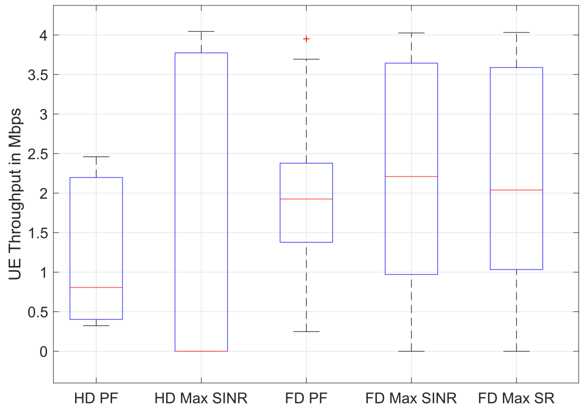

FIGURE 5. Effect of scheduling on UE throughput.

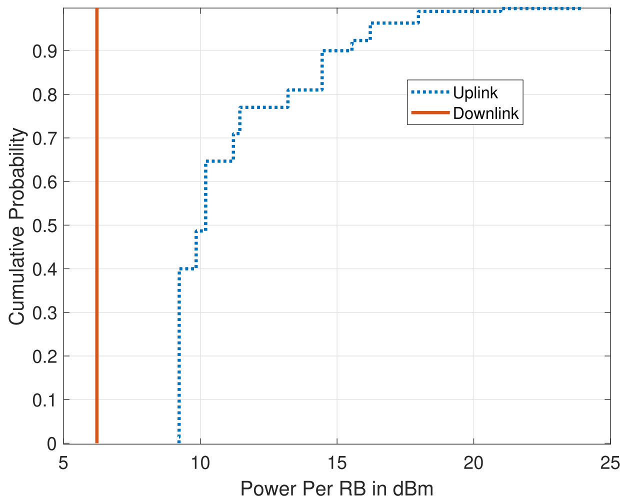

FIGURE 6. Power consumption per RB for the greedy game.

Max SINR produces a median UE throughput value of 0 Mbps with more than half the UEs attaining a throughput equal to 0 Mbps. This is in comparison to a median of about 2.2 Mbps for its full-duplex counterpart. Finally, in comparison to the greedy Max Sum-Rate algorithm from the state-of-the-art, our full-duplex Max SINR algorithm produces similar maximum and minimum throughput values with a slightly better median value (2.2 to 2.05 Mbps).

C. POWER CONSUMPTION

For all the following simulation and analysis, our full-duplex Proportional Fair scheduler is used to allocate the radio resources to pairs of uplink-downlink UEs. Our game proposals are used for power allocation.

1) THE GREEDY GAME

Allocating power on the RBs using the greedy game results in maximum power consumption. The resulting transmit power per RB can be seen in the cumulative distribution function (CDF) plot of Fig. 6.

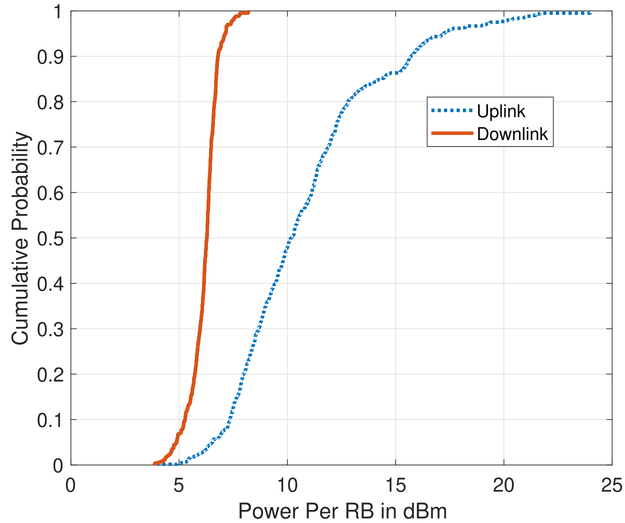

FIGURE 7. Power consumption per RB for the interference aware game.

On the downlink, the power per RB is equal to the maximum available power divided by the total number of available resources. On the uplink, the transmit power on each RB is equal to the maximum UE transmit power divided by the number of RBs allocated to a certain UE. The transmit power on the RBs in the downlink is approximately 6.2 dBm, and for the uplink UEs it ranges between 8 and 24 dBm.

2) THE INTERFERENCE AWARE GAME

Similarly, allocating power on the RBs using the interference aware game also leads to maximum power usage. Nonetheless, the power is not equally divided on the RBs as before. Figure 7 has a CDF plot of the power allocated per RB on the uplink and the downlink.

On the downlink the power on the RBs varies between 4 and 8 dBm, and on the uplink it varies between 4 and 24 dBm. This variation comes as a result of including the generated interferences in the corresponding player utilities. This will also result in better throughput values for the UEs as we later on illustrate.

3) THE ENERGY EFFICIENT GAME

When we simulated this game, neither the UEs on the uplink, nor the BS on the downlink consumed maximum power. In fact, in more than 97% of the cases, the power assigned on each RB defaults to the minimum allowed power at about 6 dBm. As we show later on, this power level was enough to produce good throughput results, alongside the added benefit of enhancing the energy efficiency, the objective of this game.

Nonetheless, the lower power limit is not the only factor impacting the allocation process. For example, if we increase the cell radius to 1 km, the game will result in higher power values on the RBs in order to improve the player utilities. This can be seen in Fig. 8. In this case, the power is significantly increased. This is due to two main reasons. First, to compensate the SINR losses resulting from increased BS-UE distances, and second in response to the now

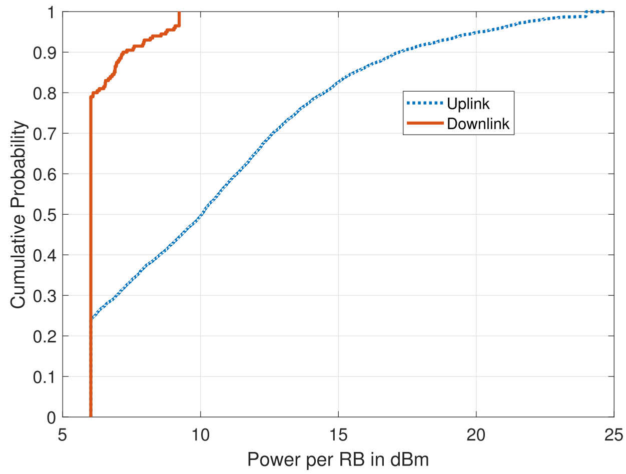

FIGURE 8. Power consumption per RB for the energy efficient game in a large cell scenario.

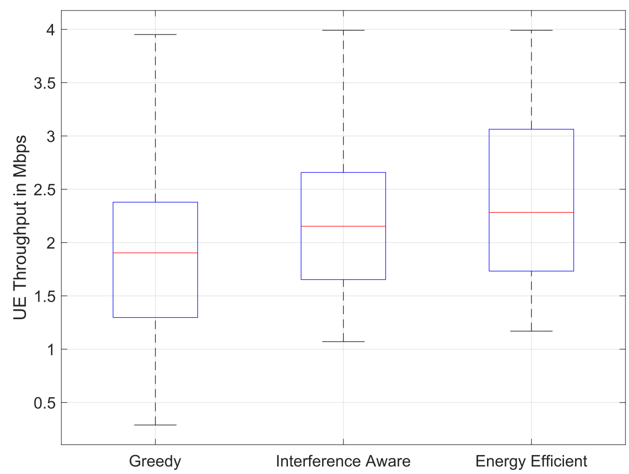

FIGURE 9. Effect of power allocation on UE throughput in a small cell scenario.

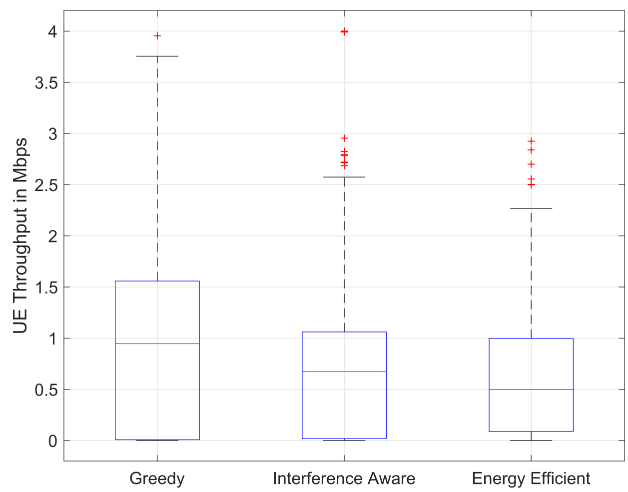

FIGURE 10. Effect of power allocation on UE throughput in a large cell scenario.

decreased UE-UE interferences as a result of the cell size being increased. The power per RB on the uplink now has a median value at 10 dBm and can reach up to 24 dBm. On the downlink, about 20% of the UEs now transmit at a power larger than the minimum value.

D. PERFORMANCE EVALUATION IN TERMS OF UE THROUGHPUT

We assess the performance of our proposed power allocation algorithms in terms of resulting UE throughput. We simulate the power allocation proposals alongside the full-duplex Proportional Fair scheduling algorithm. The cell radius considered in this simulation is 120 m. The results can be seen in Fig. 9.

The greedy game, the interference aware game, and the energy efficient game are all plotted. Comparing between the greedy game and the interference aware game, it is evident that better results are produced when the players mind the interferences they generate. The interference aware game has a higher maximum throughput value of 4 Mbps and a higher minimum value as well (1.2 Mbps compared to 0.25 Mbps for the greedy game). Nonetheless, it is clear that both algorithms waste power. The simulations for the energy efficient game produce good results with lower power consumption. In comparison, the greedy game had around 25% of the UEs with throughput values less than 1.3 Mbps, about the minimum recorded throughput value for the energy efficient game.

Nonetheless, the relevance of one game over the other might change depending on the scenario at hand. In what follows, we increase the cell radius to 1 km, and note the resulting UE throughput values for each of the games. The results can be seen in Fig. 10. The minimum power per RB is set to 6 dBm.

An increase in cell size decreases inter-UE interference and lowers the SINR values at UEs far away from the BS. An increase in UE power per RB would most benefit the UEs as it would improve their SINR, without affecting other UEs as much as it did before. As a result, the greedy game now produces the best performance in terms of UE throughput values. It produces a higher median value at about 1 Mbps, compared to 0.65 and 0.5 Mbps for the other two games, and it has much more UEs attaining throughput values close to the demand of 4 Mbps as well.

E. PERFORMANCE EVALUATION IN TERMS OF AVERAGE UE WAITING DELAY

As our queue model is non full-buffer, we are able to compute the average UE waiting delay using Little's formula. We calculate the latter across multiple simulation runs for full-duplex Proportional Fair alongside each of our power allocation proposals. We also compute the average waiting delay for the full-duplex Max Sum-Rate algorithm simulated using the greedy game, and for half-duplex Proportional Fair using maximum power allocation as well. In this simulation,

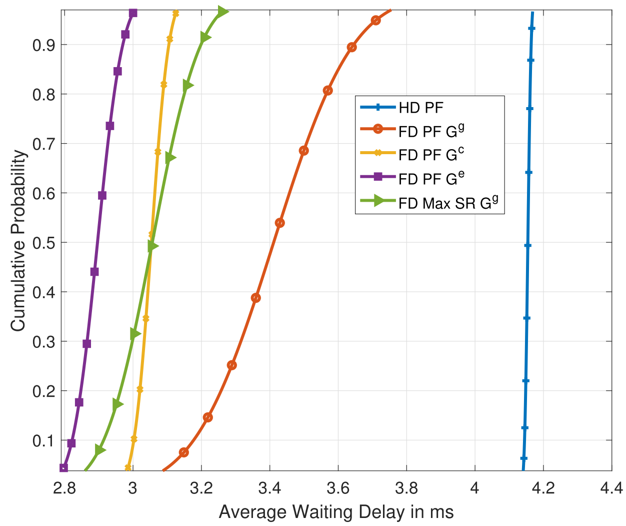

FIGURE 11. Effect of power allocation on average UE waiting delay in a small cell scenario.

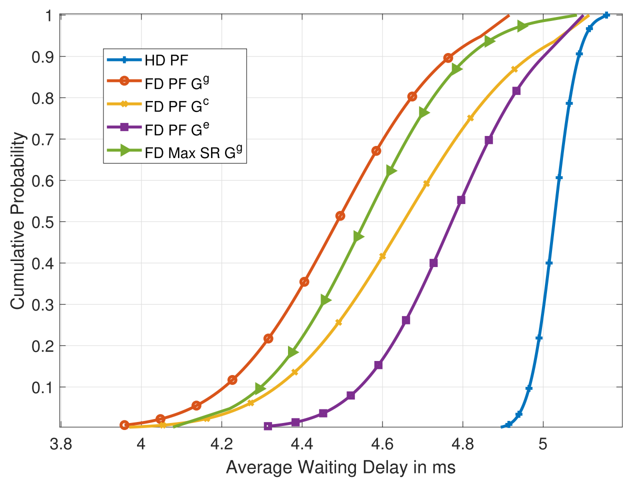

FIGURE 12. Effect of power allocation on average UE waiting delay in a large cell scenario.

the cell has a radius of 120 m. The results can be seen in the CDF plot of Fig. 11.

As with the UE throughput values, the half-duplex algorithm produces the worst waiting delays with averages exceeding 4.16 ms. The greedy game produces average UE waiting delays between 3.1 and 3.75 ms, the interference aware game between 3 and 3.1 ms, and the energy efficient game between 2.8 and 3 ms. On the other hand, the greedy Max Sum-Rate algorithm results in average UE waiting delays ranging between 2.9 and 3.3 ms. As with throughput, the fairness imposed by Proportional Fairness scheduling will come at a cost in network and UE performances. Nonetheless, in this case both the interference aware and the energy efficient game were able to outperform, or at least match, the performance of greedy scheduling.

We now repeat the same simulation, albeit with the cell radius increased to 1 km. Fig. 12 has a CDF plot with the resulting average UE waiting delays. Similar to the

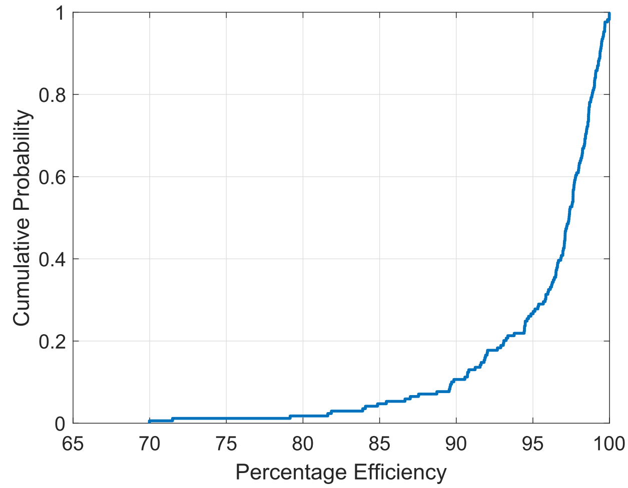

FIGURE 13. The price of anarchy for the interference aware game.

results seen in the previous section, the performances of the games are flipped. While half-duplex Proportional Fair still produces the largest average waiting delay (about 5 ms), the greedy game now actually produces the lowest waiting delay with average values between 3.95 and 4.85 ms. In this case, the greedy game produces lower waiting delays than the greedy Max Sum-Rate scheduler. This can be traced back to several factors, primarily the ability of full-duplex Proportional Fair to further exploit the effects of multi-user diversity in a dynamic arrivals scenario.

F. THE PRICE OF ANARCHY

The price of anarchy [28] is a game theory concept which measures how the efficiency of a system degrades due to the selfish behavior of its players. We study the price of anarchy in the case of the interference aware game. In this non-cooperative game proposal the uplink UEs and the BS on the downlink will each seek to maximize their own utilities. This is done in turn until an NE is achieved. A more global approach would be to maximize the sum of the uplink and downlink utilities. This problem can be written as follows:

$$\max_{\boldsymbol{P}_\gamma} \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{D}}\sum_{k\in\mathcal{K}}\Big(\log\Big(\frac{P_{ik}h_{ik}}{N_{0k}+\frac{P_{0k}}{SIC}+P_{ik}h_{ij,k}}\Big) + \log\Big(\frac{P_{0k}h_{0k}}{N_{jk}+P_{ik}h_{ij,k}+\frac{P_{0k}}{SIC}}\Big)\Big), \quad (28a)$$

$$\text{subject to} \sum_{k\in\mathcal{K}} P_{\gamma k} \le p_\gamma^{max}, \quad (28b)$$

$$P_{\gamma k} \ge p_\gamma^{min}, \quad \forall k \in \mathcal{K}, \quad (28c)$$

where (28a) is the objective of this problem: to maximize the sum of the player utilities. (28b) and (28c) are the power constraints for the BS on the downlink and the UEs on the uplink, for $\gamma = 0$ and $\gamma = i$, respectively.

In what follows we compare the resulting objective value from the sum of maximizing the separate utilities (distributed approach) vs. the maximization of the sum of utilities *i.e.*, the global optimal (centralized approach). Figure 13 has a CDF plot of the sum of objective values generated by maximizing the player utilities separately divided by the result yielded by the global problem.

In more than 80% of the cases, the selfishness of the non-cooperative game costs less than 7% in objective efficiency. In some rare cases, the selfish approach achieves less than 80% of the result achieved by the global objective. Nonetheless, this disparity in objectives does not yield better throughput results for the global optimal problem. As the uplink and downlink transmissions are intertwined, an increase in the uplink UE power will negatively impacts its paired downlink UE and vice versa.

XI. CONCLUSION

In this paper, we put forward a game theoretic framework for power allocation in full-duplex wireless networks. Coupled with both greedy and fair scheduling algorithms, we propose several non-cooperative games for power allocation. These games are played between the user equipment on the uplink and the base station on the downlink. The first of these games is greedy, wherein each player seeks to maximize its radio conditions individually. The second game is interference aware, wherein players take their generated interference into account. And our third proposal, the energy efficient game, aims to better utilize the available power while combating the full-duplex interferences. Via a set of simulations we show that the relevance of each game depends in fact on the scenario at hand. The energy efficient game saves power and is most viable in small cells, whilst the greedy game delivers the most in terms of performance when it comes to large cells. In future works, we examine the efficiency of our work in a multi-cell network.

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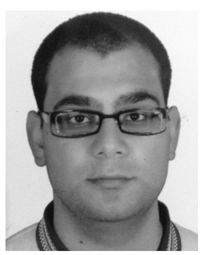

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