A Statistical Model for Uplink/Downlink Intercell Interference and Cell Capacity in TDD HetNets

Bachir Lahad, Marc Ibrahim and Samer Lahoud
Saint Joseph University of Beirut
Faculty of Engineering, ESIB
Mar Roukos, Lebanon

Kinda Khawam
University of Versailles
Versailles, France

Steven Martin
University of Paris Sud
LRI Laboratory
Orsay, France

***Abstract*—Time-division duplexing (TDD) systems can allow a dynamic adjustment of uplink and downlink resources according to the time-variant traffic loads. The interference generated by the concurrent uplink and downlink transmission in different cells, brings a new challenge to interference modeling in cellular networks. In this paper, an analytical framework is developed to evaluate the performance of a TDD system in heterogeneous networks (HetNets) which considers a concurrent uplink and downlink transmission in two different types of cells, macro and small cells. Firstly, we derive an analytical expression for the distribution of the interferer location considering all possible interference scenarios that could occur in TDD-based networks while taking into account the harmful impact of interference. Secondly, based on the latter result, we derive the distribution and moment generating function (MGF) of the uplink and downlink inter-cell interference considering a network consisting of one macro cell and one small cell. Finally, we build on the derived expressions to analyze the average capacity of the reference cell in both uplink and downlink transmissions. Monte-Carlo simulation results are provided to demonstrate the accuracy of the derived analytical expressions, in various realistic scenarios.**

***Index Terms*—Wireless communications, cellular networks, interference statistics, HetNets, TDD, spectral efficiency, MGF, distance distribution.**

I. Introduction

The rapid growth in wireless data traffic and bandwidth-intensive services (video, live streaming, etc.) necessitates finding viable solutions to improve services quality and maximize the network performance. To deal with this issue, HetNets [1] were introduced in 3GPP as one of the new features to meet these advanced requirements. Because of the difference in uplink (UL) and downlink (DL) traffic loads expected in the next HetNets generation, it becomes essential to dynamically adjust UL/DL resources. To support this new approach, dynamic time-division duplexing (TDD) ([2], [3]) has been proposed.

Several network performance metrics can be studied and statistically modeled to analyze a TDD based HetNet. One important metric and key performance factor in cellular networks is the Inter-Cell Interference (ICI). Statistical modeling of ICI plays an imperative role in evaluating the system performance metrics and developing efficient interference mitigation techniques for 5G networks. Deriving closed form expressions for ICI helps designers in developing and evaluating advanced enhancement techniques, and reduces the need for time consuming Monte-Carlo simulations.

A. Related Work

Several recent studies considered the modeling of ICI where closed-form formulas are derived to compute network performance. Some of these studies tackled individually the downlink case, other works considered the uplink case.

For the downlink case, a semi-analytical distribution for the signal-to-interference-noise ratio (SINR) has been derived in [4] under path loss and log normal shadowing for femtocell networks. In [5], the applicability of the Gaussian and binomial distributions for modeling the downlink ICI is investigated. In [6], an analytical approach based on geometric probability was developed for downlink performance analysis of a HetNet network model where the coverage probability and the spectral efficiency have been derived and verified by simulation. A novel circular interference model was introduced in [7] to facilitate statistical analysis in networks with regular grid layout. The key idea was to spread the power of the interferers uniformly along the circumcircle of the grid-shaping polygon.

Several research works for the uplink appear in [8],[9] and [10]. In [11], a new approach based on Gaussian approximation was introduced to analyze the uplink signal to interference ratio (SIR) performance. In [12], the scheduling-based interference models for LTE networks was considered where the distance distribution and thus the interference distribution was derived for different resource allocation schemes such as proportional fair, greedy and round robin scheduling schemes. In [13], uplink capacity (in bps/Hz) was derived with closed-form expressions in both dedicated and shared spectrum access scenarios. The proposed framework exploited the distance distributions based on geometric probability theory to characterize the co-tier and cross-tier uplink interferences.

As for the modeling of interference in TDD systems, the work in [14] introduced a statistical framework to analyze uplink/downlink interactions where analytical expressions for the four interference types in TDD systems were derived. For interference-aware scenarios, the distribution of the distance between the source and victim of interference was based on the scheduling algorithms defined in [12]. The distance distribution probability in [14] was scheduling based instead of geometric probability based. Moreover, it did not evaluate important network performance metrics such as average cell capacity. Instead, interference maps were generated to analyze the impact of TDD operation in a given network.

B. Contributions and Organization

In this paper, we consider analyzing both uplink/downlink interactions, whereas other works introduced either a downlink ICI model as in [5] and [6] or an uplink ICI model as in [12] and [13]. Few studies considered modeling a TDD system as in [14], however they do require the knowledge of instantaneous scheduling decisions and did not evaluate a network performance indicator. In this paper, operating a TDD-based HetNet network necessitates four interference scenarios depending mainly on the geographical distance between the neighboring cell and the reference cell. In addition to the statistical framework developed to model the downlink and the uplink interferences, closed expressions are derived to measure the average cell capacity. Finally, the impact of the transmission power, path loss and the small cell location, on both metrics (interference and capacity) is investigated in detail and validated using Monte-Carlo simulations.

The remainder of the paper is organized as follows. In section II, novel closed-form expressions for the statistics of uplink/downlink interferences are derived. In section III, we derive the probability density function (PDF) and the MGF of the signal of interest for both downlink and uplink scenarios. Based on this, in section IV, we derive the average downlink/uplink network capacity. Finally, Section V presents selected numerical and simulation results followed by concluding remarks in Section VI.

II. Statistical Interference Modeling

In order to characterize the statistics of each interference scenario that could occur in TDD-based HetNets, the proposed framework mandates analyzing in details both the downlink and uplink interferences. In the coming sections, the radius of the macro cell and the small cell will be denoted by R_m and R_s respectively. The distance between macro and small cell will be designated by d .

A. Downlink Interference Distribution

In this section, we derive the PDF $f_I(x)$ of the downlink interference at a reference small cell s inflected by the macro neighboring cell m using *cells intersections* method. Two cases should be considered if dynamic TDD configuration was adopted between the macro and the small cells:

1. Calculate the downlink interference at small cell when the macro cell is in downlink mode: Down - Down mode.
2. Calculate the downlink interference at small cell when the macro cell is in uplink mode: Up - Down mode.

1) Down - Down mode: The downlink interference from the macro BS to the mobile user associated with a small BS (BS-MS interference) is defined as follows:

$$I = KP_m r_{ms}^{-\gamma}, \tag{1}$$

where γ is the path-loss exponent, r_{ms} is the distance of the user from the macro BS m, P_m denotes the transmit power of the macro BS per channel, and K the composite fading

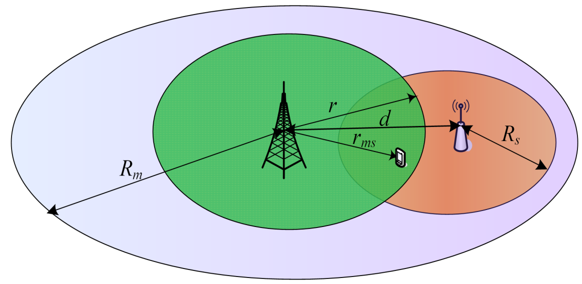

Fig. 1. Down - Down mode

channel.
We start by computing the CDF of r_{ms} as follows:
$P(r_{ms} < r) = \frac{\text{Intersection area between C(0,0) of radius } r \text{ and C(d,0) of radius } R_s}{\text{Area of C(d,0) of radius } R_s}$
as shown in Fig. 1,

$$F_{r_{ms}}(r) = P(r_{ms} < r) = \frac{R_s^2 \arccos(\frac{d^2+R_s^2-r^2}{2dR_s}) + r^2 \arccos(\frac{d^2+r^2-R_s^2}{2dr})}{\pi R_s^2} - \frac{1/2\sqrt{4d^2r^2-(d^2-R_s^2+r^2)^2}}{\pi R_s^2}.$$

A simplified expression for the PDF of r_{ms} can therefore be obtained, by doing some algebraic manipulations, as follows:

$$f_{r_{ms}}(r) = \frac{1}{\pi R_s^2}(\pi r - 2r \arcsin(\frac{d^2+r^2-R_s^2}{2dr})). \tag{2}$$

When considering the composite fading K as a constant, the cumulative distribution function (CDF) of interference I can be formulated as:

$$F_I(x) = 1 - F_{r_{ms}}\Big((\frac{x}{C})^{-\frac{1}{\gamma}}\Big). \tag{3}$$

Applying chain rule, the PDF can be written as:

$$f_I(x) = \frac{1}{I\gamma} f_{r_{ms}}\Big((\frac{x}{C})^{-\frac{1}{\gamma}}\Big)(\frac{x}{C})^{\frac{-1}{\gamma}}, \tag{4}$$

where $C = KP_m$.
Next, we will derive a closed-form expression for the MGF of the interference based on (4) as follows:

$$M_I(t) = \int_0^\infty e^{-tx} f_I(x) dx. \tag{5}$$

2) Up - Down mode: The downlink interference from the macro user to the small cell user (MS-MS interference) is defined as follows:

$$I = KP_{um} r_0^{-\gamma}, \tag{6}$$

where r_0 is the distance of the small cell user from the macro user and P_{um} denotes the transmit power of the macro user per channel.
The CDF of r_0 can be derived as:

$$F_{r_0}(r) = \int F_{r_0}(r)_{|r_{sm}} f_{r_{sm}}(r_{sm}) dr_{sm}, \tag{7}$$

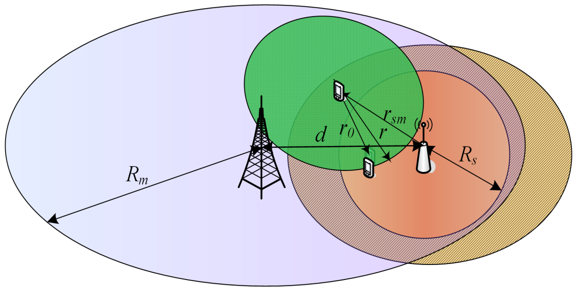

Fig. 2. Up - Down mode

$F_{r_0}(r)_{|r_{sm}}$ denotes the CDF of r_0 conditioned on r_{sm} which is considered as constant. $F_{r_0}(r)$ in (7) is derived by averaging over the PDF of r_{sm}.

$f_{r_{sm}}(r_{sm})$ can be deducted from $F_{r_{sm}}(r_{sm})$:

$$F_{r_{sm}}(r_{sm}) = \begin{cases} \frac{\pi r_{sm}^2 - \pi R_s^2}{\pi R^2 - \pi R_s^2}, & R_s \leq r_{sm} < R - d \\ H(r_{sm}), & R - d \leq r_{sm} \leq R + d, \end{cases}$$

$$\text{where } H(r_{sm}) = \frac{1}{\pi R^2 - \pi R_s^2}\Big(R^2 \arccos(\frac{d^2 + R^2 - r_{sm}^2}{2dR}) + r_{sm}^2 \arccos(\frac{d^2 + r_{sm}^2 - R^2}{2dr_{sm}}) - 1/2\sqrt{4d^2 r_{sm}^2 - (d^2 - R^2 + r_{sm}^2)^2} - \pi R_s^2\Big).$$

Then $f_{r_{sm}}(r_{sm})$ can be written as:

$$f_{r_{sm}}(r_{sm}) = \begin{cases} \frac{1}{\pi R^2 - \pi R_s^2}(2\pi r_{sm}), & R_s \leq r_{sm} < R - d \\ v(r_{sm}), & R - d \leq r_{sm} \leq R + d, \end{cases} \tag{8}$$

where $v(r_{sm}) = \frac{1}{\pi R^2 - \pi R_s^2}(\pi r_{sm} - 2r_{sm} \arcsin(\frac{d^2 + r_{sm}^2 - R^2}{2dr_{sm}}))$.

Since r defined values will vary based on r_{sm} values, $F(r_0)_{|r_{sm}}$ is given as follows:

$$F_{r_0}(r)_{|r_{sm}} = \begin{cases} 0, & r < r_{sm} - R_s \\ G(r), & r - R_s \leq r_{sm} \leq r + R_s \\ 1, & r > r_{sm} + R_s, \end{cases} \tag{9}$$

where

$$G(r) = \frac{R_s^2 \arccos(\frac{r_{sm}^2 + R_s^2 - r^2}{2r_{sm}R_s}) + r^2 \arccos(\frac{r_{sm}^2 + r^2 - R_s^2}{2r_{sm}r})}{\pi R_s^2} - \frac{1/2\sqrt{4r_{sm}^2 r^2 - (r_{sm}^2 - R_s^2 + r^2)^2}}{\pi R_s^2}.$$

In this context, $F_{r_0}(r)$ can be derived from (7), (8) and (9) as follows:

$$F_{r_0}(r) = 0P(r_{sm} > r + R_s) + 1P(r_{sm} < r - R_s) + \int_{r-R_s}^{r+R_s} F_{r_0}(r)_{|r_{sm}} f_{r_{sm}}(r_{sm}) dr_{sm},$$

$$F_{r_0}(r) = F_{r_{sm}}(r - R_s) + \int_{r-R_s}^{r+R_s} F_{r_0}(r)_{|r_{sm}} f_{r_{sm}}(r_{sm}) dr_{sm}, \tag{10}$$

given that the lowest and the highest values of r_{sm} are respectively R_s and R+d.

The same logic applies for the PDF:

$$f_{r_0}(r) = \int_{r-R_s}^{r+R_s} f_{r_0}(r)_{|r_{sm}} f_{r_{sm}}(r_{sm}) dr_{sm}, \tag{11}$$

where $f_{r_0}(r)_{|r_{sm}} = \frac{1}{\pi R_s^2}(\pi r - 2r \arcsin(\frac{r_{sm}^2 + r^2 - R_s^2}{2r_{sm}r}))$ and $f_{r_{sm}}(r_{sm})$ as defined in (8).

Consequently, the PDF of interference I can be written as:

$$f_I(x) = \frac{1}{I\gamma} f_{r_0}\Big((\frac{x}{C})^{-\frac{1}{\gamma}}\Big)(\frac{x}{C})^{\frac{-1}{\gamma}}, \tag{12}$$

where $C = KP_{um}$.

B. Uplink Interference Distribution

In this section and similar to the downlink case, we derive the PDF $f_I(x)$ of the uplink interference in two different scenarios: Up - Up and Down - Up modes.

1) Up - Up mode: The uplink interference from the macro user to the small cell BS (MS-BS interference) is defined as follows:

$$I = KP_{um} r_{sm}^{-\gamma}, \tag{13}$$

where r_{sm} is the distance of the small cell BS from the macro user and P_{um} denotes the transmit power of the macro user per channel.

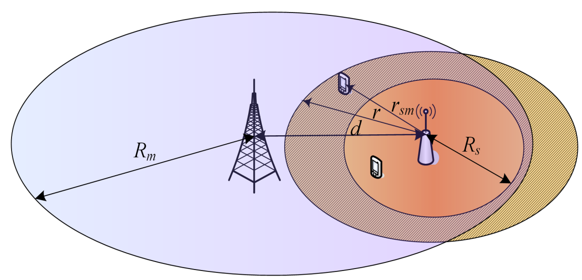

Fig. 3. Up - Up mode

For $R - d \leq r \leq R + d$, $P(r_{sm} < r)$ can be written as (see Fig. 3 for illustration):

$$P(r_{sm} < r) = \frac{\text{Area 1}}{\text{Area 2}},$$

where Area 1 = (Intersection area between C(d,0) of radius r and C(0,0) of radius R) – (Area of C(d,0) of radius R_s)

and

Area 2 = Area of C(0,0) of radius R - Area of C(d,0) of radiusR_s

$$P(r_{sm} < r) = \frac{R^2 \arccos(\frac{d^2+R^2-r^2}{2dR}) + r^2 \arccos(\frac{d^2+r^2-R^2}{2dr})}{\pi R^2 - \pi R_s^2} \frac{-1/2\sqrt{4d^2r^2 - (d^2 - R^2 + r^2)^2} - \pi R_s^2}{\pi R^2 - \pi R_s^2}.$$

For $R_s \leq r < R - d$, $P(r_{sm} < r)$ is given as:

$P(r_{sm} < r) = \frac{\pi r^2 - \pi R_s^2}{\pi R^2 - \pi R_s^2}$.

Combining the two cases:

$$F_{r_{sm}}(r) = \begin{cases} \frac{\pi r^2 - \pi R_s^2}{\pi R^2 - \pi R_s^2}, & R_s \leq r < R - d \\ H(r), & R - d \leq r \leq R + d, \end{cases} \tag{14}$$

$$\text{where } H(r) = \frac{R^2 \arccos(\frac{d^2+R^2-r^2}{2dR}) + r^2 \arccos(\frac{d^2+r^2-R^2}{2dr})}{\pi R^2 - \pi R_s^2} \frac{-1/2\sqrt{4d^2r^2 - (d^2 - R^2 + r^2)^2} - \pi R_s^2}{\pi R^2 - \pi R_s^2}.$$

$f_{r_{sm}}(r)$ can be derived from the CDF as follows:

$$f_{r_{sm}}(r) = \begin{cases} \frac{1}{\pi R^2 - \pi R_s^2}(2\pi r), & R_s \leq r < R - d \\ s(r), & R - d \leq r \leq R + d, \end{cases} \tag{15}$$

where $s(r) = \frac{1}{\pi R^2 - \pi R_s^2}(\pi r - 2r \arcsin(\frac{d^2+r^2-R^2}{2dr}))$.

The CDF of I can be formulated as:

$$F_I(x) = 1 - F_{r_{sm}}\Big((\frac{x}{C})^{-\frac{1}{\gamma}}\Big). \tag{16}$$

Applying chain rule, the PDF of I can be written as:

$$f_I(x) = \frac{1}{I\gamma} f_{r_{sm}}\Big((\frac{x}{C})^{-\frac{1}{\gamma}}\Big)(\frac{x}{C})^{\frac{-1}{\gamma}}, \tag{17}$$

where $C = KP_{um}$.

2) Down - Up mode: The uplink interference from the macro BS to the small cell BS (BS-BS interference) is defined as follows:

$$I = KP_m d^{-\gamma}, \tag{18}$$

where d is the distance of the small cell BS from the macro BS and P_m denotes the transmit power of the macro BS per channel.
In this case, I is a constant. Accordingly, the MGF of I will be defined as:

$$M_I(t) = \mathbb{E}[e^{(-tI)}] = e^{-tI} \tag{19}$$

where $I = KP_m d^{-\gamma}$ (see Fig. 4 for illustration).

Table I summarizes different distance statistics of each interference scenario.

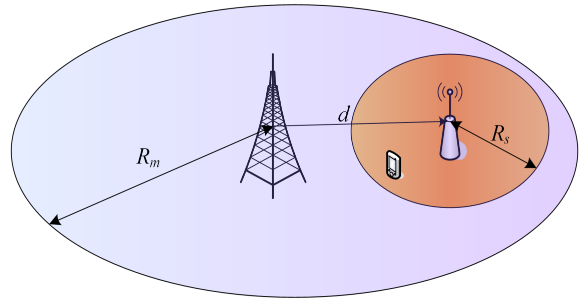

Fig. 4. Down - Up mode

TABLE I
DISTANCE DISTRIBUTION PER INTERFERENCE SCENARIO

Macro Cell	**Small Cell**	**Interference Scenario**	**PDF/MDF**
Down	Down	BS-MS	Equation (2)
Up	Down	MS-MS	Equation (11)
Up	Up	MS-BS	Equation (15)
Down	Up	BS-BS	Equation (19)

III. PDF AND MGF OF THE SIGNAL OF INTEREST

A. Downlink transmission

The signal power received by a randomly selected user at the reference small cell on a downlink channel is defined as follows:

$$S = KP_s r_s^{-\gamma}, \tag{20}$$

where r_s is the distance of the user from the small BS s and P_s denotes the transmit power of the small BS per channel.

Since we consider uniformly distributed users, the distribution of the distance r_s of a small cell user is given by:

$$f_{r_s}(r) = \frac{2r}{R_s^2}, \quad 0 \leq r \leq R_s. \tag{21}$$

When considering the composite fading K as a constant, the PDF of the received signal S can be formulated as:

$$f_S(x) = \frac{1}{x\gamma} f_{r_x}\Big((\frac{x}{C})^{-\frac{1}{\gamma}}\Big)(\frac{x}{C})^{\frac{-1}{\gamma}}, \tag{22}$$

where $C = KP_s$.
By replacing (21) in (22), the PDF of the received signal S can be written as:

$$f_S(x) = \beta x^{-(\frac{2+\gamma}{\gamma})}, \tag{23}$$

where $\beta = \frac{2C^{\frac{2}{\gamma}}}{\gamma R_s^2}$.

Consequently, the MGF $M_S(t) = \mathbb{E}[e^{(-tS)}] = \int_0^\infty e^{-tx} f_S(x)dx$ can be given as:

$$M_S(t) = \int_a^\infty \beta x^{-(\frac{2+\gamma}{\gamma})} e^{-tx} dx, \tag{24}$$

where $a = CR_s^{-\gamma}$.

B. Uplink transmission

When the small cell is operating in uplink mode, the signal power received at the reference small cell on an uplink channel from a randomly selected user is defined as follows:

$$S = KP_{us}r_s^{-\gamma}. \tag{25}$$

Same formulas applied to the PDF and MGF of the signal of interest while changing the variable C to $C = KP_{us}$. P_{us} denotes the transmit power of small cell user per channel.

IV. Analytical Evaluation of Ergodic Capacity

The average capacity per unit bandwidth can be calculated using the lemma proposed in [15] as follows:

$$C = \mathbb{E}[\ln(1+\tfrac{S}{I+\sigma^2})] = \int_0^\infty \tfrac{M_I(t)-M_S(t)M_I(t)}{t} e^{-(\sigma^2)t}dt$$
$$= \int_0^\infty \tfrac{M_I(t)-M_S(t)M_I(t)}{te^{-at}} e^{-(\sigma^2+a)t}dt,$$

where S and I considered independent for all cases. This expression can be solved efficiently by expressing it in terms of the weights w_e and abscissas x_e of a Laguerre orthogonal polynomial as follows:

$$C = \sum_{e=1}^{n} w_e \frac{M_I(x_e/\sigma^2) - M_S(x_e/\sigma^2)M_I(x_e/\sigma^2)}{x_e}$$

$$C = \sum_{e=1}^{n} w_e \Big(\frac{M_I(x_e/(\sigma^2+a))}{x_e e^{-(ax_e)/(\sigma^2+a)}} - \frac{M_S(x_e/(\sigma^2+a))M_I(x_e/(\sigma^2+a))}{x_e e^{-(ax_e)/(\sigma^2+a)}} \Big). \tag{26}$$

The a parameter was introduced and fine tuned to meet the best convergence level of Laguerre expression. It will vary depending on the MGF function defined for each interference scenario.

V. Numerical and Simulation Results

In this section, we first define the input parameters and discuss the Monte-Carlo simulation results that are provided to demonstrate the accuracy of the derived analytical expressions. Our objective is to analyze analytically the performance of a TDD-based HetNet and compare the results with Monte-Carlo simulations output. The main performance metric that is being measured is the average capacity of a small cell user per unit bandwidth (C_s). Uplink and downlink throughputs have been evaluated and averaged over a defined TDD frame.

A. Input Parameters

The radius of the macro and small cell is taken as 1000 m and 200 m, respectively while considering uniformly distributed users in both cells. The path-loss exponent is $\gamma = 2$ and the composite fading is set as constant. We consider thermal noise power variance as 10^{-12} W/Hz and transmission power per sub-channel as $P_m = 40$ W for macro BS and $P_s = 0.25$ W for small BS. However, a control of the user uplink transmission power is considered within the below presented scenarios. Note that the distance between macro and small cells is set to $d = 800$ m. Without loss of generality, we consider frame structures where all uplink/downlink combinations, between the macro and small cells, are taken into account as shown in Table II. The Monte Carlo simulation results are averaged over 10 000 iterations.

TABLE II
Frame Structures in Macro and Small Cells

Macro cell	DL	DL	UL	UL
Small cell	DL	UL	DL	UL

B. Analytical and Simulation Results

Figure 5 captures the decrease in C_s by increasing the small cell radius. This is due to the fact that small cell users are in average closer to the macro BS and thus the interference from the macro BS and macro users will be more significant. Moreover, we can observe that numerical results are in close agreement with the Monte-carlo simulation.

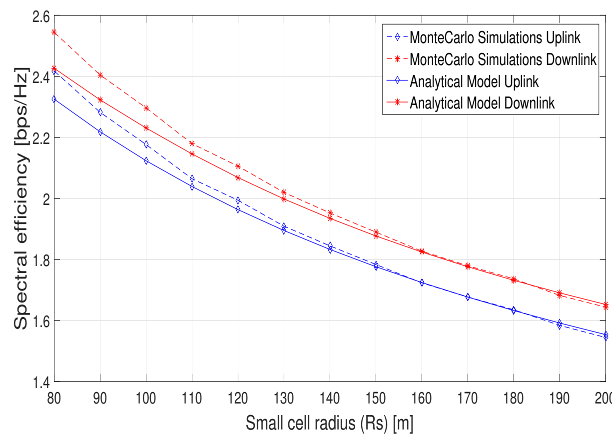

Fig. 5. Capacity per unit bandwidth in a small cell as a function of small cell radius (R_s)

Figure 6 captures the increase in C_s when moving away the small cell from the macro cell. The interference, more specifically triggered by the macro BS, degrades when increasing the distance between the small and the macro cell.

Figure 7 depicts C_s for different values of macro user uplink transmission power. Note that the increase of P_{um} value reduces the average small cell capacity and this is due to the increase of interference triggered mainly by the macro users.

Figure 8 investigates the effect of increasing the path loss exponent. It can be observed that this increase enhances the performance of the small cell users in uplink and downlink by mitigating the effect of the interference signal.

It can be concluded that the derived capacity expression in (26) matches nearly perfectly Monte Carlo simulation results. Moreover, when evaluating the capacity expression in (26) over 4 TDD sub frames (Table II), the execution time was found to be reduced by more than 50% comparing to the time

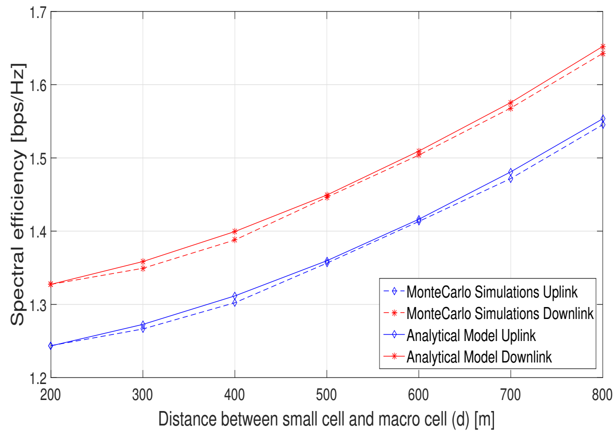

Fig. 6. Capacity per unit bandwidth in a small cell as a function of the distance between macro and small cells (d)

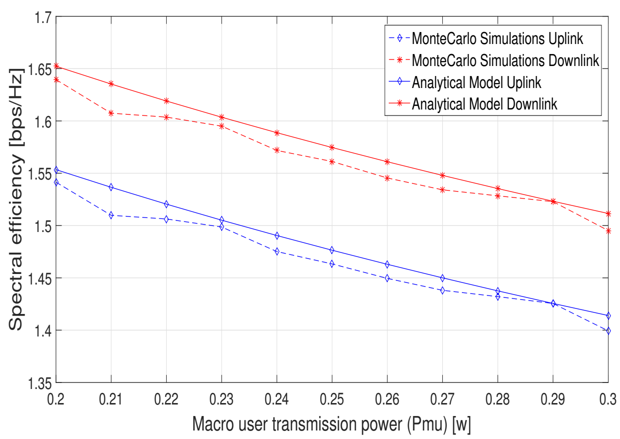

Fig. 7. Capacity per unit bandwidth in a small cell as a function of the macro user transmission power (P_{um})

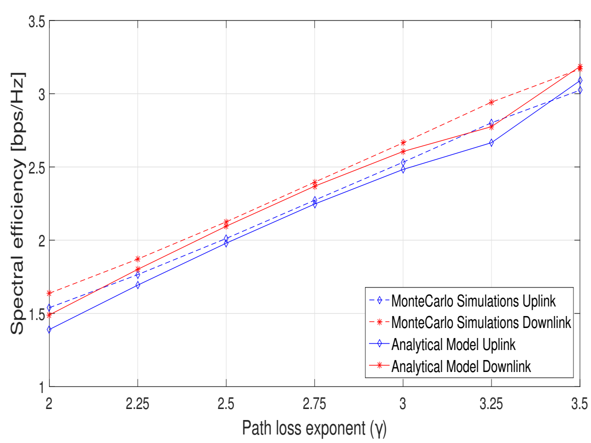

Fig. 8. Capacity per unit bandwidth in a small cell as a function of the path loss exponent (γ)

required to run 10 000 Monte-Carlo simulations over the same period.

VI. Conclusion

Statistical modeling of ICI is a key factor in the design and implementation of wireless cellular networks. It plays an important role in the establishment of advanced interference mitigation techniques. In this work, we have proposed an uplink and a downlink inter-cell interference model for a dynamic TDD HetNet system considering various distance distributions. The proposed model provides a valuable tool to evaluate the system performance in terms of downlink/uplink average capacity. The provided numerical results help in optimizing the design parameters under various conditions. For example, the system performance was evaluated when controlling the macro user transmission power. Further, the effect of large path loss exponents proved to make small cell users perform even better. The derived expressions can be also considered to evaluate other performance metrics such as network outage and user fairness. This new approach can be extended to include the decoupled downlink and uplink access and various composite fading channel models.

References

[1] S.Brueck, "Heterogeneous networks in LTE-Advanced, 8th International Symposium on Wireless Communication Systems," *Aachen*, pp. 171–175, 2011.

[2] "Radio frequency (RF) system scenario (release 10)," 3GPP TS 36.211, Jun. 2012.

[3] "Further enhance- ments to LTE Time Division Duplex (TDD) for Downlink-Uplink (DL-UL) interference management and traffic adaptation ," 3GPP TR 36.828, Jun. 2012.

[4] K. W. Sung, H. Haas, and S. McLaughlin. "A semi-analytical PDF of downlink SINR for femtocell networks," *EURASIP J. Wireless Commun. and Networking*, Jan. 2010.

[5] S. Plass, X. G. Doukopoulos, and R. Legouable. "Investigations on link-level inter-cell interference in OFDMA systems," *in Proc. 2006 IEEE Symposium on Communications and Vehicular Technology*, pp. 49-52.

[6] X. Yang and A. O. Fapojuwo. "Analysis of heterogeneous cellular network with hexagonal tessellated macrocells and randomly positioned small cells," *IEEE WCNC*, 2016.

[7] M. Taranetz and M. Rupp. "A circular interference model for wireless cellular networks," *International Wireless Communications and Mobile Computing Conference*, 2014.

[8] S. Elayoubi, B. Haddada, and B. Fourestie. "Performance evaluation of fre- quency planning schemes in OFDMA based networks," *IEEE Trans.Wireless Commun.*, vol. 7, no. 5, pp. 1623-1633, May 2008.

[9] R. Kwan and C. Leung. "On collision probabilities in frequency-domain scheduling for LTE cellular networks," *IEEE Commun. Lett.*, vol. 15, no. 9, pp. 965- 967, Sep. 2011.

[10] I. Viering, A. Klein, M. Ivrlac, M. Castaneda, and J. A. Nossek. "On uplink intercell interference in a cellular system," *in Proc. 2006 IEEE International Conference on Communications*, pp. 2095-2100.

[11] M. Ding, D. Lopez-Perez, G. Mao, and Z. Lin. "DNA-GA: A new approach of network performance analysis," *Proc. IEEE ICC 2016*,arXiv:1512.05429 [cs.IT], to be published.

[12] H. Tabassum, Z. Dawy, E. Hossain, and M. S Alouini. "A framework for uplink intercell interference modeling with channel-based scheduling," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 206-217, Jan. 2013.

[13] H. Tabassum, Z. Dawy, E. Hossain, and M. S Alouini. "Interference statistics and capacity analysis for uplink transmission in two-tier small cell networks: A geometric probability approach," *IEEE Trans. Wireless Commun.*, vol. 13, no. 7, pp. 3837-3852, Jul. 2014.

[14] Ahmad El-Hajj, Naeem Akl, Bilal Hammoud, and Zaher Dawy. "On interference modeling for the analysis of uplink/downlink interactions in TDD-OFDMA networks," *2015 International Wireless Communications and Mobile Computing Conference (IWCMC)*, pp. 497-502, 2015.

[15] K. A. Hamdi. "A useful lemma for capacity analysis of fading interference channels," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 411-416, Feb. 2010.