A Game Theoretic Approach for Power Allocation in Full Duplex Wireless Networks

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***Abstract*—The benefits of full-duplex wireless communications, specifically with respect to their half-duplex counterparts, are now well under examination. As a result, research into the corresponding scheduling and power allocation algorithms has thrived. In this paper, we propose a non-cooperative game theoretic algorithm for power allocation in full-duplex orthogonal frequency division multiple access networks. The game is played between user equipment on the uplink, and the base station on the downlink. The objective of the game is two-fold: maximizing the signal-to-noise-plus-interference ratio, while hindering the harmful interferences resulting from full-duplex operation. We prove that our game is super-modular. For such a game, a best response algorithm is capable of attaining a Nash equilibrium. We simulate our proposal along with a fairness based scheduling algorithm and show that it improves user equipment throughput and reduces the waiting delay.**

I. Introduction

With the rapidly increasing user equipment (UE) throughput requirements [1], and the constant addition of connected devices, the race for the next big wireless network breakthrough is at full throttle. Full-duplex (FD) wireless technologies, made feasible by the introduction of self-interference cancellation (SIC) techniques, are quite capable of meeting this booming demand. A full-duplex orthogonal frequency division multiple access (FD-OFDMA) network is a wireless system model within which the base station (BS) operates in FD, and the UEs remain half-duplex (HD). This keeps the intricacies of implementing FD communications at the BS. Two UEs, one transmitting and one receiving, use the same radio resource, on which the BS transmits and receives concurrently. The two UEs are said to be paired on the radio resource. A theoretical doubling of the capacity is challenged by two repercussions of FD transmissions: self-interference, and intra-cell co-channel interference.

The first, self-interference, is experienced at the BS and thus affects uplink UEs in the network. It is the interference inflicted by the transmitted, and relatively large, signal on the feeble signal being received on the same radio resource. This interference is battled using SIC techniques [2], a set of analog and digital technologies now well capable of canceling up to 130 dB of signal interference.

The second, intra-cell co-channel interference, results from two UEs using the same radio resources within the same cell. It degrades the performance of downlink UEs in the network. It is up to the scheduler to deal with co-channel interference, by selecting pairs of UEs which exhibit minimum interference. As a result, scheduling on the uplink and the downlink can no longer be done independently as in typical HD networks.

Due to the added FD interferences, both scheduling and power allocation play a vital role in enhancing the performance of an FD system. Current radio resource and power allocation schemes, designed for HD networks as in [3] and [4], benefit from orthogonal downlink and uplink channels and can thus be optimized independently. In contrast, in the context of FD wireless communications, the optimization of scheduling and power allocation has to be done jointly for the uplink and the downlink because of the concept of pairing and the generated FD interferences. Consequently, it is not possible to apply any traditional HD scheduling or power allocation algorithms to FD networks in a straightforward manner.

In this paper, we propose a game theoretic approach to power allocation in FD-OFDMA wireless networks. We aim to improve UE signal-to-noise-plus-interference (SINR) ratio, and at the same time, curb the interferences each UE inflicts on its FD pair. Our game is non-cooperative by design, it does not necessitate a central authority capable of enforcing any set of rules aimed at battling the network interferences. This choice reduces signaling and processing requirements that otherwise would be necessary. Nonetheless, if UEs on the uplink and the downlink take their decisions utterly independently, the result would be maximum interference. As such, we implement seperate utilities for the uplink players and for the BS on the downlink. These utilities take the interferences each player generates into consideration.

We prove that our game is super-modular. The latter is known to attain a pure Nash equilibrium via a best response algorithm. Numerical solutions are provided for maximizing player utilities. We simulate our proposal along with a fairness based scheduling algorithm and show that it improves UE throughput values and reduces the waiting delay.

The rest of this paper is structured as follows. Section II discusses the related works. Section III presents the system model. As power allocation is done for UEs already scheduled, we need a scheduling algorithm to fully assess our devised power allocation proposal. To this aim, section IV puts forward a fair scheduling algorithm we previously proposed, and used to allocate resources in our simulations. In section V, we detail our game theoretic approach to power allocation in FD-OFDMA wireless networks. Simulations and results are portrayed in section VI, and finally the paper is concluded in section VII.

II. Related Works

In this section, we look at the related works in the state-of-the-art. In the scope of FD communications, it is important to consider two major pillars: scheduling and power allocation.

The papers in [5]–[7] are among the earliest in verifying the validity of FD communications. The authors in these works proposed FD network models, studied the limitations of FD communications, and went as far as implementing FD modules in their aim to assess achievable gains.

More recent works in the state-of-the-art moved towards scheduling and power allocation. The articles in [8]–[11] focus on greedy scheduling algorithms coupled with optimal power allocation mechanisms. The objectives in these papers are centered around maximizing the network sum-rate. In some, heuristic algorithms were proposed to replace the mathematically intractable joint problem.

Probably the closest to our objective in this article are the papers in [12] and [13]. The authors in [12] introduce the idea of using game theory in the context of FD operations. Their article surveys possible applications and implementations of game theory in relation to scheduling and power allocation in different FD network scenarios. In [13], the authors use a game theoretic approach for resource allocation in FD networks. They couple their algorithm with a water-filling based power allocation problem and iterate till a Nash equilibrium is achieved. They show that their proposal is profitable with respect to HD networks.

In this paper, we propose a game theoretic approach to power allocation in FD-OFDMA wireless networks, and couple it with a scheduler that seeks fairness in resource allocation between UEs. Our game is non-cooperative. It seeks to maximize UE SINR, as well as curb network interferences. We show that our game is super-modular and hence best response dynamics converge to a Nash equilibrium. Our system model uses a non-full buffer traffic model, unlike the vast majority of the state-of-the-art [8]–[13]. Non-full buffer traffic, like streaming and video, would make up to 78 % of the global mobile traffic by the year 2021 [1], highlighting the importance of studying the implications of such traffic models. A dynamic traffic model additionally allows us to compute packet level metrics such as the waiting delay. Furthermore, by using a distributed approach, and by implementing separate utilities for each set of players, our algorithm is significantly less complex than the mixed integer non-linear problems proposed in the articles mentioned above. This makes it easier to implement in real life scenarios.

III. System Model

A. Radio Model

We consider a single-cell FD-OFDMA network. This network is comprised of a full-duplex BS, and half-duplex UEs. The UEs are virtually divided into two sets: an uplink UE set, denoted by $\mathcal{I}$ and a downlink UE set, denoted by $\mathcal{D}$. The scheduler will pair between uplink and downlink UEs on the radio resources. This network is illustrated in Fig. 1.

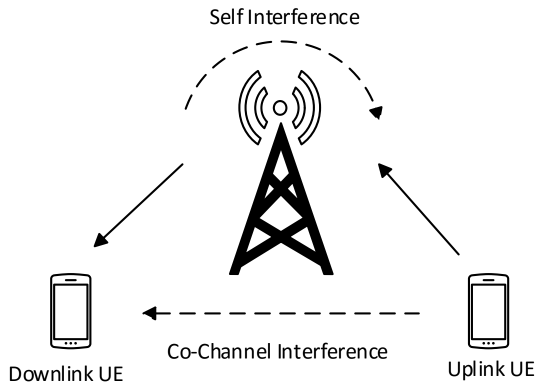

Figure 1. Network model and interferences

In our work, we assume that the physical layer is operated using an OFDMA structure. The radio resources are divided into time-frequency resource blocks (RBs). In the time domain, an RB contains an integer number of OFDM symbols. In the frequency domain, an RB contains adjacent narrow-band subcarriers and experiences flat fading. Scheduling decisions for downlink and uplink transmissions are made in every transmission time interval (TTI). At the beginning of each TTI, $\mathcal{K}$ RBs are to be allocated. The TTI duration is chosen to be smaller than the channel coherence time. With these assumptions, UE radio conditions will vary from one RB to another, but remain constant over a TTI. The modulation and coding scheme (MCS), that can be assigned to a UE on an RB, depends on its radio conditions. For performance evaluation, we consider LTE-like specifications, with an RB being composed of 12 subcarriers and 7 OFDM symbols.

An adapted formula is used to calculate the SINR that takes into consideration the co-channel interference between a UE pair, and the self-interference cancellation performed by the BS. The SINR of uplink UE i observed on RB k, whilst paired with downlink UE j, is expressed as:

$$S_j^u(i,k) = \frac{P_{ik}h_{ik}}{N_{0k} + \frac{P_{0k}}{SIC}}, \quad (1)$$

where on RB k, P_{ik} is the power emitted by UE i, h_{ik} is the channel gain between uplink UE i and the BS, and P_{0k} is the power emitted on the downlink by the BS. SIC denotes the self-interference cancellation performed by the BS, and thus $\frac{P_{0k}}{SIC}$ is the residual self-interference. Finally, N_{0k} is the noise power at the BS on RB k. Furthermore, the SINR observed by downlink UE j alloted RB k, and paired with uplink UE i, is expressed as:

$$S_i^d(j,k) = \frac{P_{0k}h_{jk}}{N_k^d + P_{ik}h_{ji,k}}, \quad (2)$$

where h_{jk} is the channel gain between downlink UE j attributed RB k and the BS, and $h_{ji,k}$ is the channel gain between downlink UE j attributed RB k and interfering UE i, matched on that same RB. As such, $P_{ik}h_{ji,k}$ is the co-channel interference affecting downlink UE j. Finally, N_k^d is the noise power at downlink UE j allocated RB k.

B. Channel State Information

The state of a wireless channel is determined by the combined effect of several factors, the most pertinent of which, are the path loss, the shadowing, and the fast fading. Knowledge of the channel on a certain wireless link permits adapting the transmission to the communication channel. This is essential in achieving reliable communications, and for making efficient resource allocation decisions.

Full duplex communications add to the complexity of determining the channel states. In FD systems, additional information on the channel in between the UEs of a certain pair is required. In our work, we statistically model the inter-UE channel as follows:

$$h_{ji,k} = G_t G_r L_p A_s A_f \quad (3)$$

G_t and G_r are the antenna gains at the transmitter and the receiver, respectively. L_p represents the path loss, or equivalently the mean attenuation the signal undergoes in this channel. A_s and A_f are two random variables that respectively represent the shadowing effect, and the fast fading effect. In this work, the scheduler is assumed to have perfect channel state information. The impact of imperfect channel state information on FD network performances has been investigated in our previous work in [14].

C. Traffic Model

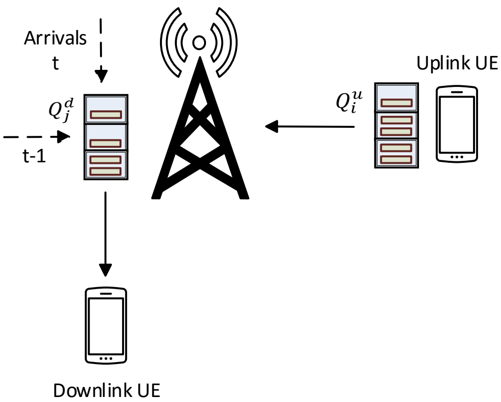

Figure 2. Traffic model: UE pair i-j

Our scheduling is queue-aware (Fig. 2). Each UE has a predefined throughput demand which determines the rate at which the UE will transmit or receive. A downlink UE has a queue at the BS, denoted Q_j^d, that it wants to receive. An uplink UE has a queue of bits it wants to transmit to the BS, denoted Q_i^u. UE queues are updated each TTI. They are filled according to a random process with a number of bits/s equal, on average, to the UE throughput demand. Once the scheduling is done for a certain TTI, the scheduler computes the number of bits each UE can transmit or receive, and the UE queues are deducted accordingly. Any bits remaining in a UE queue at the end of a TTI are carried on to the next.

IV. FD Proportional Fair Scheduling

In this paper, we utilize a scheduling algorithm we previously proposed [14]. The aim is to allocate the RBs in a manner that maximizes the system throughput, while at the same time insures a certain level of fairness. To this end, we proposed an FD Proportional Fair algorithm, which allocates RBs to the pairs of UEs with the highest sum of priorities. The priority of a UE is a function of its current radio conditions, represented by the number of bits a UE can transmit, or receive, on the current RB, and its historic radio conditions, represented by the number of bits it has already transmitted. The priority for an uplink UE i, paired with a downlink UE j on RB k, for example, is defined as:

$$\rho_j(i,k) = \frac{T_{ijk}^u}{T_i}, \quad (4)$$

where T_i is the number of bits UE i has transmitted over a certain time window, and T_{ijk}^u is the number of bits UE i can transmit on RB k while paired with UE j. The optimization problem for FD Proportional Fair is presented below, where the objective function is to maximize the sum of priorities $i.e.$, select the pairs with the highest priorities. The UE pair-resource assignment variable z_{ijk}, is defined $\forall\ k \in \mathcal{K}$, $\forall\ i \in \mathcal{I}$, $\forall\ j \in \mathcal{D}$, and is equal to one if uplink UE i is paired with downlink UE j on RB k. It is equal to zero otherwise. The resource allocation problem for every TTI t is formulated as follows.

(P_s^t):

$$\text{Maximize} \quad \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{D}} z_{ijk}(\rho_j(i,k) + \rho_i(j,k)), \quad (5a)$$

$$\text{subject to} \quad \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{D}} z_{ijk} \le 1,\ \forall k \in \mathcal{K}, \quad (5b)$$

$$\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{D}} z_{ijk} T_{ijk}^u \le D_i^u,\ \forall i \in \mathcal{I}, \quad (5c)$$

$$\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}} z_{ijk} T_{ijk}^d \le D_j^d,\ \forall j \in \mathcal{D}, \quad (5d)$$

$$z_{ijk} \in \{0,1\},\ \forall i \in \mathcal{I}, \forall j \in \mathcal{D}, \forall k \in \mathcal{K}. \quad (5e)$$

Similar to before, T_{ijk}^d is the number of bits UE j can receive on RB k while paired with UE i. T_{ijk}^u and T_{ijk}^d depend mainly on the radio conditions of the UEs. In addition, D_i^u is the demand of uplink UE i $i.e.$, the number of bits in its queue. Likewise, D_j^d is the demand of downlink UE j.

Equation (5a) is the objective of our problem, to select the pairs which have the highest sum of priority values. According to the constraint (5b), each RB should be allocated to either one or no pair. Equations (5c) and (5d) dictate the efficiency of the resource allocation process. They ensure that no UE will get resources more than it needs to transmit or receive the entirety of its queue.

V. Non-Cooperative Game for Power Allocation

Non-Cooperative game theory models the interactions between players competing for a common resource. It does not necessitate any central authority or signaling between the players. Hence, it is well adapted to power control modeling in an FD setting. Following the SINR formulas for uplink (1) and downlink (2) UEs, an increase in the power of an uplink UE will increase its SINR but at the same time cause added interference on its paired downlink UE. Vice-versa, an increase in the transmit power at the BS, would increase the SINR of the receiving downlink UE, but cause added interference on the paired uplink UE. UEs on the uplink and the BS on the downlink, *i.e.,* the decision makers, are playing for contradicting objectives. As such, we define a multi-player game $\mathcal{G}$ between the BS (coined player 0) and the $|\mathcal{I}|$ uplink UEs. In particular, on every alloted RB k, uplink UE i will compete with the BS. The formulation of this non-cooperative game $G = \langle M, S_0 \times \prod_i S_i, U \rangle$ can be described as follows:

- A finite set of players $M = (BS, UE\, i)$ paired on the same RB k. In fact, on each allocated RB k, a two-players game is engaged between the BS and uplink UE i matched on RB k.
- The action of a given player is the amount of power allocated on RB k, the strategy chosen by the BS is then $\boldsymbol{P}_0 = (P_{01}, ..., P_{0|\mathcal{K}|})$ and the strategy chosen by any uplink UE i is $\boldsymbol{P}_i = (P_{i1}, ..., P_{i|\mathcal{K}|})$. A *strategy profile* $\boldsymbol{P} = (\boldsymbol{P}_0, \boldsymbol{P}_1, ..., \boldsymbol{P}_{|\mathcal{I}|})$ specifies the strategies of all players.
- For the BS, the space of pure strategies is S_0 given by what follows:

$$S_0 = \{\boldsymbol{P}_0 \in \mathbb{R}^{|\mathcal{K}|}, \text{such as} \sum_{k \in \mathcal{K}} P_{0k} \leq p_0^{max} \text{ and } P_{0k} \geq p_0^{min}, \forall k \in \mathcal{K}\}$$

- For each uplink UE i, the space of pure strategies is S_i given by what follows:

$$S_i = \{\boldsymbol{P}_i \in \mathbb{R}^{|\mathcal{K}|}, \text{such as} \sum_{k \in \mathcal{K}} P_{ik} \leq p_i^{max} \text{ and } P_{ik} \geq p_i^{min}, \forall k \in \mathcal{K}^i\},$$

  where $\mathcal{K}^i$ is the set of RBs allocated to UE i and $S = S_0 \times S_1 \times ... \times S_{|\mathcal{I}|}$ is the set of all strategies;
- A set of utility functions $U = (U_0, U_{i \in \mathcal{I}})$ that quantify players' profit for a given strategy profile.

Note that an uplink player i will not transmit on an RB it was not allocated.

A. Player Utilities

In this game, each player takes into account its harmful interfering impact on its adversary. Since the game is non-cooperative, it is necessary that each player minds the interferences they generate. If these interferences are not accounted for in the utilities, each player will seek to maximize its own gains independently, and consequently, increase its transmit power. This would generate maximum interference in the network. Let $j(i,k)$ be a reference to downlink UE j paired with uplink UE i on RB k as a result of scheduling. For simplicity, in the remainder of this paper we use $j = j(i,k)$. The utility of every uplink UE i is thereafter written as:

$$U_i = \sum_{k \in \mathcal{K}^i} \log\left(\frac{P_{ik} h_{ik}}{N_{0k} + \frac{P_{0k}}{SIC} + P_{ik} h_{ji,k}}\right). \quad (6)$$

As for the BS:

$$U_0 = \sum_{k \in \mathcal{K}} \log\left(\frac{P_{0k} h_{jk}}{N_k^d + P_{ik} h_{ji,k} + \frac{P_{0k}}{SIC}}\right), \quad (7)$$

where $\mathcal{K}^i$ is the set of RBs scheduled to UE i. The SINR for the UEs, on the uplink and on the downlink, are thus inherently included. Additionally, the co-channel interference, which degrades the performance of downlink UEs, is now also affecting the utility of uplink UEs. The self-interference, which degrades the performance of uplink UEs, is now also affecting the utilities relating to downlink UEs. As such, we can seek to improve UE performance, while at the same time account for the resulting interferences. Via our simulations, we show that our proposed utilities converge to an efficient Nash equilibrium which improves UE performance.

B. A Super-modular Game

In a non-cooperative game, a valid solution is one where all players adhere to a Nash equilibrium, which is a profile of strategies in which no player will profit by deviating its strategy unilaterally. A Nash equilibrium is a static concept that often abstracts away the question of how it is reached. Thus, the main challenge in non-cooperative game theory is to devise practical algorithms to reach such an equilibrium. The simplest example of such algorithms are repeated best response dynamics: each player selects the best (locally optimal) response to other players' strategies, until convergence. However, convergence of repeated best response is not guaranteed in general. For this game, we are in presence of a type of games called super-modular, where a best response algorithm permits attaining Nash equilibriums. In what follows, we introduce a formal definition of super-modular games and prove that our power allocation game belongs to the latter class. According to [15], $\mathcal{G}$ is super-modular if for any player $\gamma \in M$:

1) The strategy space S_γ is a compact sub-lattice of $\mathbb{R}^{|\mathcal{K}|}$;
2) The objective function is super-modular, that is $\frac{\partial^2 U_0}{\partial P_0 \partial P_i} \geq 0$ and $\frac{\partial^2 U_i}{\partial P_i \partial P_0} \geq 0$ $\forall i \in \mathcal{I}$, $\forall \boldsymbol{P} \in S$, and $\forall k \in \mathcal{K}$.

In [15], [16], proof is given for the following two results in a super-modular game:

- If each player γ either initially uses its lowest or largest policy in S_γ, then a best response algorithm converges monotonically to a Nash equilibrium.
- If we start with a feasible policy, then the sequence of best responses monotonically converges to a Nash equilibrium: it monotonically increases in all components in the case of maximization in a super-modular game.

Proposition 5.1: Game $\mathcal{G} \langle M, S_0 \times \prod_i S_i, U \rangle$ is a super-modular game.

Proof: To prove the super-modularity of the game, we need to verify the aforementioned conditions. First, the strategy space S_γ is obviously a compact convex set of $\mathbb{R}^{|\mathcal{K}|}$. Hence, it suffices to verify the super-modularity of the objective function U_γ of any player γ as there are no constraint policies for $\mathcal{G}$. For any uplink UE i, we have:

$$\frac{\partial^2 U_i}{\partial P_{ik} \partial P_{0k}} = \frac{\frac{h_{ji,k}}{SIC}}{(N_{0k} + \frac{P_{0k}}{SIC} + P_{ik} h_{ji,k})^2} \geq 0, \quad (8)$$
$$\forall i \in \mathcal{I}, \forall k \in \mathcal{K}.$$

And for the BS, we have what follows:

$$\frac{\partial^2 U_0}{\partial P_{0k}\partial P_{ik}} = \frac{\frac{h_{ji,k}}{SIC}}{(N_k^d + \frac{P_{0k}}{SIC} + P_{ik}h_{ji,k})^2} \geq 0, \quad (9)$$
$$\forall i \in \mathcal{I}, \forall k \in \mathcal{K}.$$

□

C. Computing the Nash Equilibrium

As we proved that we are in presence of a supermodular game, we implement a best response algorithm to reach its pure Nash equilibrium. At the convergence of the best response algorithm, the Nash equilibrium is the solution of the following two optimization problems:

$$\max_{\boldsymbol{P}_\gamma} U_\gamma(\boldsymbol{P}_\gamma, \boldsymbol{P}_{-\gamma}) \quad (10a)$$
$$\text{subject to} \quad \sum_{k\in\mathcal{K}} P_{\gamma k} \leq p_\gamma^{max}, \quad (10b)$$
$$P_{\gamma k} \geq p_\gamma^{min}, \; \forall k \in \mathcal{K}. \quad (10c)$$

where p_γ^{max} (resp. p_γ^{min}) is the maximal (resp. minimal) power limit on the uplink for $\gamma \in \mathcal{I}$ and on the downlink for $\gamma = 0$. As the optimization problems in (10) are convex, the Karush-Kuhn-Tucker (KKT) conditions enable determining a global optimal (*i.e.,* the Nash equilibrium at convergence) [17]. The KKT conditions associated with $P_{\gamma k}, \forall k \in \mathcal{K}$ gives what follows:

$$\frac{1}{P_{\gamma k}^*} - \frac{1}{b_{\gamma k} + P_{\gamma k}^*} = \lambda_\gamma, \; \forall k \in \mathcal{K} \quad (11a)$$
$$\lambda_\gamma \times (p_\gamma^{max} - \sum_{k\in\mathcal{K}} P_{\gamma k}^*) = 0 \quad (11b)$$
$$P_{\gamma k}^* \geq p_\gamma^{min}, \; \forall k \in \mathcal{K} \quad (11c)$$
$$\lambda_\gamma \geq 0. \quad (11d)$$

where λ_γ is the KKT multiplier associated with the constraint (10b), and

$$b_{\gamma k} = \begin{cases} \frac{N_{0k} + \frac{P_{0k}}{SIC}}{h_{ji,k}}, \gamma = i \in \mathcal{I} \\ SIC \times (N_k^d + P_{ik}h_{ji,k}), \gamma = 0 \end{cases} \quad (12)$$

We deduce from (11a) that λ_γ cannot be null. As such, all $P_{\gamma k}^*$ are the solution of a second order equation that gives $P_{\gamma k}^* = \frac{b_{\gamma k}\cdot(\sqrt{1+\frac{4}{b_{\gamma k}\lambda_\gamma}}-1)}{2}$, where λ_γ can be computed numerically owing to $\sum_{k\in\mathcal{K}} P_{\gamma k}^* = p_\gamma^{max}$. Finally, in respect with constraint (11c), we have what follows for the BS:

$$P_{0k}^* = \max(p_0^{min}, \frac{SIC \times (N_k^d + P_{ik}h_{ji,k})}{2}\cdot (\sqrt{1 + \frac{4}{SIC \times (N_k^d + P_{ik}h_{ji,k})\lambda_0}} - 1)), \quad (13)$$

and for any uplink UE i:

$$P_{ik}^* = \max(p_i^{min}, \frac{N_{0k} + \frac{P_{0k}}{SIC}}{2h_{ji,k}}\cdot(\sqrt{1 + \frac{4}{\frac{N_{0k}+\frac{P_{0k}}{SIC}}{h_{ji,k}}\lambda_i}} - 1)). \quad (14)$$

Algorithm 1 has the scheduling and power allocation algorithm. The matrix p^u has the transmit power of every uplink UE on every RB it was allocated. p^d has the BS transmit power on all the RBs. Typically, the algorithm will reach a Nash equilibrium for the power allocation step in 3 to 4 iterations.

Algorithm 1 Scheduling and Power Allocation Algorithm

1: **Requires:** Maximum tolerance $\epsilon \geq 0$.
2: **Input:** UE radio conditions, channel states, initial power settings p_0^u and p_0^d .
3: **For** TTI t=1....T
4: **Step 1: Scheduling**
5: RBs are allocated following (P_s^t) in (5)
6: **Step 2: Power Allocation**
7: **Repeat:**
8: Solve (10) in the uplink $\forall\ i \in \mathcal{I}$.
9: Update p_n^u
10: Solve (10) in the downlink for the BS
11: Update p_n^d
12: δ^d= $\|p_n^d - p_{n-1}^d\|$, δ^u= $\|p_n^u - p_{n-1}^u\|$
13: $n \leftarrow n+1$
14: **Until** $\delta^d \leq \epsilon$ and $\delta^u \leq \epsilon$
15: **End For**

VI. Simulations and Results

A. Simulation Parameters

We seek via our different simulation scenarios to address the gains attributed to our game theoretic proposal. The simulation parameters we used are presented in Table I.

Table I
Simulation Parameters

Parameter	Value
Cell Specifications	Single-Cell, 120 m Radius
Number of RBs	60
Maximum BS/UE Transmit Power	24 dbm
SIC Value	10^{11}
Number of UEs	20 UEs: 10 downlink, 10 uplink
UE Distribution	Uniform
Demand Throughput	4 Mbps
Fast Fading	Rayleigh. σ=1
Shadowing	Normal law. μ=0 dB σ^2=10 dB
Path Loss Model	Extended Hata Path Loss Model

The channel gain takes into account the path loss, the shadowing and the fast fading effects. The path loss is calculated using the extended Hata path loss model [18]. The shadowing is modeled by a log-normal random variable $A_s = 10^{(\frac{\xi}{10})}$, where ξ is a normal distributed random variable with zero mean and standard deviation equal to 10. The fast fading is modeled by an exponential random variable A_f with unit parameter. This model is used for urban zones and it takes into account the effects of diffraction, reflection and scattering caused by city structures.

B. Power Consumption

In this section, we observe how our proposal allocates power on the RBs. Figure 3 has a cumulative distributive frequency (CDF) plot with results. On the downlink, the power on the RBs varies between 4 and 8 dBm,

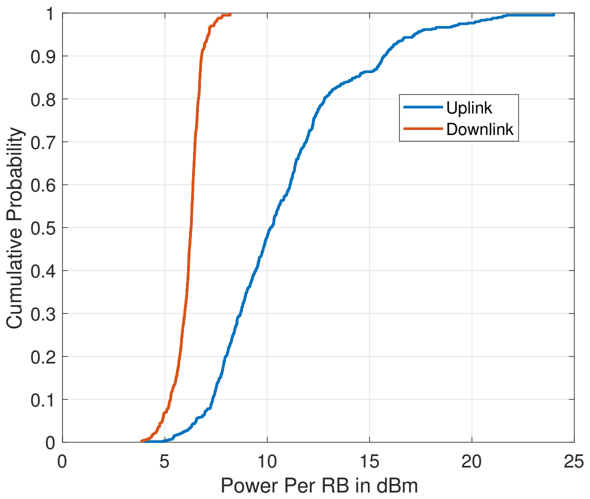

Figure 3. Power consumption per RB

and on the uplink it varies between 4 and 24 dBm. This variation comes as a result of the proposed player utilities. If both uplink and downlink power levels were to be simultaneously increased, the generated interferences would also be maximized.

C. Effect on UE Throughput

We study the effect of our power allocation algorithm on the performance of the UEs in the network. We simulate our scheduling algorithm (FD Proportional Fair) using first our proposed power allocation algorithm, and second using equal maximum power allocation. The latter indicates that for uplink UEs, each UE will transmit with all 24 dBm available power divided equally on the RBs it got. On the downlink, the maximum transmit power is divided equally on all the RBs. For reference, an HD Proportional Fair algorithm and the greedy FD Max Sum-Rate algorithm presented by the authors in [8], are also simulated. Equal maximum power allocation is used for HD Proportional Fair as well as FD Max Sum-Rate. Figure 4 has a CDF plot with the results.

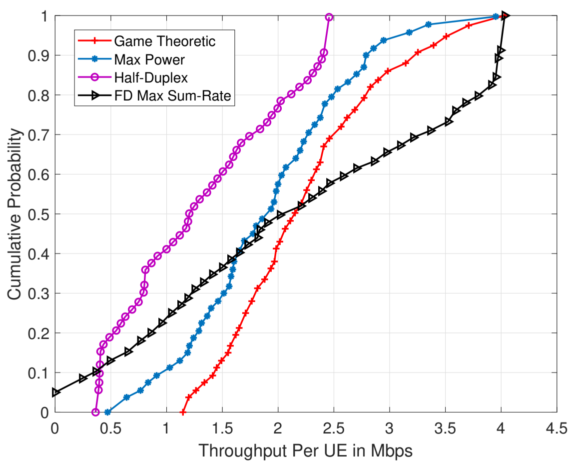

Figure 4. Effect of power allocation on UE throughput

The gains that FD operation brings is clear when comparing the HD plot with the corresponding FD plots. The plot for HD Proportional Fair lags behind the three FD plots, indicating that FD UEs almost always perform better. Furthermore, few HD UEs crossed the 2 Mbps mark (less than 20%) even though all UEs are in excellent radio conditions (HD scenario exhibits no interferences). When compared with our proposal, the worst performing FD UE, with about 1.2 Mbps throughput, faired better than more than half of the HD UEs. Comparing between our proposal and equal maximum power allocation, our power allocation algorithm results in a higher maximum throughput, close to 4 Mbps, and a higher minimum as well. Our utilities allow our algorithm to count for the intra-cell co-channel interferences. In the case of equal maximum power allocation, it is expected that the effects of FD interferences would be maximized. When the power is increased on the downlink, uplink UE performance degrades and vice-versa. Furthermore, in comparison with the Max Sum-Rate algorithm, our proposal also relatively performs better. There is nonetheless a visible contrast in scheduling objectives. FD Max Sum-Rate seeks to maximize the total network throughput, while our scheduling proposal aims to allocate resources fairly between the UEs. Hence, the lowest recorded throughput value for our proposed algorithm is close to 1.2 Mbps, compared to 0 Mbps for the FD Max Sum-Rate algorithm. Maximum power allocation on all the RBs pushes the greedy algorithm to the extremities. The performance of UEs with good radio conditions will get better, while interference on UEs with bad radio conditions is increased. As a result, HD UEs outperformed their FD Max Sum-Rate counterparts in around 10% of the simulated cases.

D. Effect on UE Waiting Delay

In this section, we study the impact of our power allocation algorithm on the transmission delay experienced by the UEs in the network. Our non-full buffer traffic model allows us to compute packet level metrics such as the waiting delay. The average UE waiting delay is calculated using Little's formula as the average queue length divided by the packet arrival rate. Figure 5 has a box plot with the average UE waiting delay in ms for each of the cases simulated in the previous section: FD Proportional Fair with game theoretic power allocation, FD Proportional Fair with equal maximum power allocation, HD Proportional Fair, and FD Max Sum-Rate with maximum power allocation.

HD UEs experience the worst delay, with a median average delay of around 4.15 ms. The greedy Max-Sum Rate algorithm UEs experience the least amount of delay with a minimum average equal to 2.85 ms and a maximum close to 3.2 ms. Our scheduling algorithm, coupled with a game theoretic approach to power allocation comes close in second. It achieves a median close to 3.05 ms with a maximum average delay of 3.15 ms. This is an impressive result considering the scheduling objective for this algorithm is fairness oriented, as compared to the greedy Max Sum-Rate algorithm. Finally, simulating our scheduling algorithm with equal maximum power allocation produced mixed results. While the algorithm still outperforms HD operation, the performance is still clearly degraded in comparison with game theoretic power allocation.

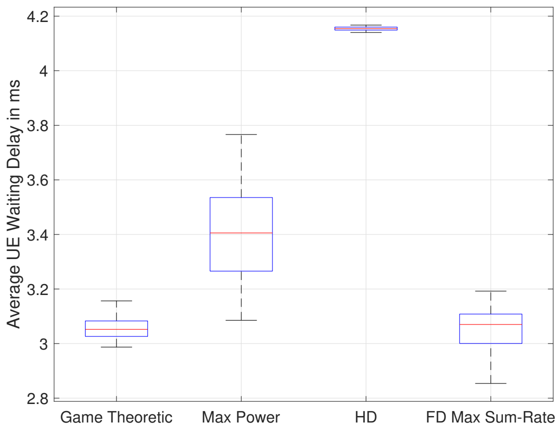

Figure 5. Effect of power allocation on UE waiting delay

E. Case of Low SIC

We aim to study the performance of our power allocation algorithm in the case of low self-interference cancellation. Following the SINR formulas, a decrease in the network ability to cancel self-interference degrades the performance of uplink UEs. We lower the SIC factor from 10^{11} to 10^{9}, and repeat the simulations.

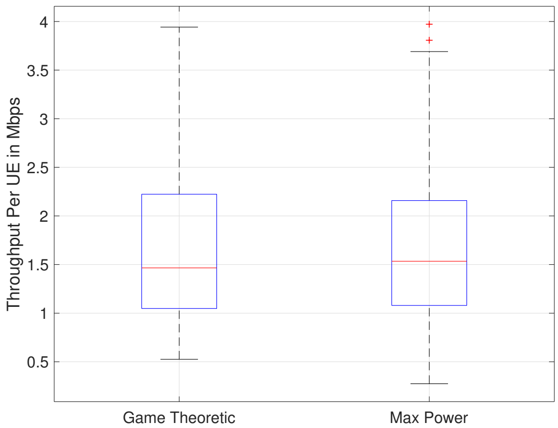

Figure 6. Effect of low SIC on UE performance

Figure 6 has a box plot of the throughput attained by the UEs as a result of scheduling using our proposed power allocations algorithm, vs. using fixed equal maximum power allocation. Power allocation with fixed maximum powers should be able to benefit uplink UEs in this situation. More power translates into higher SINR values. Nonetheless, our algorithm still outperforms it. Our algorithm shows a higher maximum, close to 4 Mbps, and a higher minimum as well, close to 0.5 Mbps. Our algorithm can better adapt to the decrease in SIC capabilities. Nonetheless, the performance of the algorithm with maximum power is closer to our proposal in this case than in the normal case. This is because the best response to lower SIC values is higher transmit powers on the uplink, something inherently present in the maximum power simulation.

VII. Conclusion

In this paper, we proposed a game theoretic approach to power allocation in FD-OFDMA wireless networks. Our game is non-cooperative. As such, and in order to take FD interferences into consideration, we propose separate utilities for the uplink and the downlink. The utilities seek to improve UE radio conditions, and at the same time, curb the interferences that each UE inflicts on its FD pair. We show that our game is super-modular, and we implement a best response algorithm to reach a Nash equilibrium. Finally, we address the performance gains of our proposed power allocation algorithm: our approach improves UE throughput and reduces UE waiting delays.

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