Achieving Spectral and Energy Efficiencies in Smart Cities Multi-cell Networks

Bilal Maaz, Kinda Khawam, Samer Lahoud, Jad Nasreddine and Samir Tohme

1 Introduction

Mobile communication is considered as one of the building blocks of smart cities, where citizens should be able to enjoy telecommunications services wherever they are and whenever they want in a secure and non-costly way. This can be done by dense deployment of broadband mobile systems such as Long Term Evolution (LTE) and its successors. This dense deployment will lead to higher energy consumption, and thus more gas emission and pollution. Therefore, it is crucial from environmental point of view to reduce the energy consumption. In this context, the focus of this chapter is to introduce radio resource management methods that increase energy efficient, and thus reduce pollution and power wastage. Most of the work that tackles the problem of energy efficiency in cellular networks considers the case of one single cell. In this chapter, we propose a game theoretical approach for the problem of energy efficiency in multicell LTE networks. We address the problem of ICIC in the downlink of LTE OFDMA-based systems, where the power level selection for frequency subcarriers is portrayed as a non-cooperative game in the context of self-organizing networks. The existence of Nash equilibriums (NEs) for the modeled game shows that stable power allocations can be reached by selfish eNBs. To attain these NEs, we propose a decentralized algorithm based on Best Response dynamics. In order to evaluate our proposal, we compare the obtained results to an optimal global Coordinated Multi-Point (CoMP) solution

J. Nasreddine Rafik Hariri University, Beirut, Lebanon

B. Maaz (\boxtimes) \cdot K. Khawam \cdot S. Tohme

University of Versailles, Versailles, France e-mail: bilal.maaz@ens.uvsq.fr

S. Lahoud University of Rennes I, Rennes, France

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where a central controller is the decision maker. Numerical simulations assessed the good performance, in terms of throughput and energy efficiency, of the proposed distributed approach in comparison with the centralized approach.

2 The Network Model

We consider an LTE network comprising a set of eNBs denoted by *J*. We focus on the downlink in this chapter. The time and frequency radio resources are grouped into time-frequency Resource Blocks (RBs). An RB is the smallest radio resource unit that can be scheduled to a mobile user. Each RB consists of N_s OFDM symbols in the time dimension and N_f sub-carriers in the frequency dimension (in LTE $N_s = 7$ and $N_f = 12$ in the most common configuration). The set of RBs is denoted by *K*, and the set of users is denoted by *I*. We consider the Single Input Single Output (SISO) technology in this chapter and we will consider Multi Input Multi Output (MIMO) technology in a future work. In the following, we make the following assumptions:

- We consider a fixed cell assignment and we don't consider mobility in our network model, each user typically compares the received signal power from each eNB and chooses to connect to the eNB with the strongest signal.
- In order to evaluate the maximum system performance, we consider permanent downlink traffic where each eNB has persistent traffic towards its users. We also assume that all RBs are assigned on the downlink at each scheduling epoch.
- We adopt the widely used Proportional Fair (PF) scheduler for serving active users. Symbols and variables used within this chapter are defined in Table 1.

The power consumption of eNB $j \in J$ is modeled as a linear function [1] of the average transmit power per site as: $p_j = p_j^1 \pi_j + p_j^0$. where p_j and π_j denote the average consumed power by eNB j and its transmit power, respectively. The coefficient p_j^1 accounts for the power consumption, that scales with the transmit power due to radio frequency amplifier and feeder losses.

Variables	Signification	Variables	Signification
J	Set of eNBs	π_{jk}	Transmit power of eNB j on RB k
Ι	Total set of users	θ_{ik}	Percentage of time user i is associated with RB k
Κ	Set of Resource blocks	G_{ijk}	Channel power gain (user i on RB k on eNB j)
N_0	Noise power	$ \rho_{ijk} $	SINR of user i associated eNB j served on RB k
<i>I(j)</i>	Set of users associated to eNB <i>j</i>	α_{jk}	Interference impact on RB <i>k</i> of eNB <i>j</i> among other eNBs

Table 1 Sets, parameters and variables in the chapter

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The coefficient p_j^0 models the power consumed independently of the transmit power due to signal processing and site cooling. The transmit power of each eNB is allocated to resource blocks serving the users in the network. The total transmit power of eNB *j* is the sum of the transmit power on each RB $k \in K : \pi_j = \sum_{k \in K} \pi_{jk}$. The total power consumed by any eNB *j* is given by:

$$P_j = p_j^1 \sum_{k \in K} \pi_{jk} + p_j^0.$$
 (1)

Given user *i* associated with eNB *j* (i.e. $i \in I(j)$), the signal-to-interferenceplus-noise-ratio (SINR) of this user when served on RB *k* is given by:

$$\rho_{ijk} = \frac{\pi_{jk} G_{ijk}}{N_0 + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}}$$
(2)

where G_{ijk} is the path gain of user *i* on resource block *k* on eNB *j* (i.e. the average path gain over the sub-carriers in the resource block), and N_0 is the noise power, which is, without loss of generality, assumed to be the same for the all users on all resource blocks.

Assuming a proportional fairness service by each eNB on each resource block, the system utility function is given by what follows:

$$U(\theta) = \sum_{j \in J} \sum_{i \in I(j)} \frac{g(|I(j)|)}{|I(j)|} \sum_{k \in K} \log(\rho_{ijk})$$

= $\sum_{j \in J} \sum_{i \in I(j)} \sum_{k \in K} \frac{g(|I(j)|)}{|I(j)|} \log\left(\frac{\pi_{jk}G_{ijk}}{N_0 + \sum_{j' \neq j} \pi_{j'k}G_{ij'k}}\right).$ (3)

where $g(|I(j)|) = \sum_{s=1}^{|I(j)|} 1/s$, as we consider the PF scheduler with a fast varying fading channel (Rayleigh fading). In the following sections we will provide solution for the problem of maximizing the above mentioned utility function.

3 Centralized Power Control Approach

We cast hereafter the centralized power control problem:

$$P(\pi) : \underset{\pi}{\operatorname{maximize}} \sum_{j \in J} \sum_{i \in I(j)} \sum_{k \in K} \frac{g(|I(j)|)}{|I(j)|} \log\left(\frac{\pi_{jk} G_{ijk}}{N_0 + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}}\right)$$
(4a)

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Subject to
$$\sum_{k \in K} \pi_{jk} \le p_j^{max}, p_j^{min} \le \pi_{jk}, \forall k \in K. \forall j \in J$$
 (4b)

Problem (4a, b) is non-linear and apparently difficult, non-convex optimization problem. However, it can be transformed into a convex optimization problem in the form of geometric programming by performing a variable change $\hat{\pi}_{jk} = \log(\pi_{jk})$ and defining $\hat{N}_0 = \log(N_0)$ and $\hat{G}_{ijk} = \log(G_{ijk})$. The resulting optimization problem deemed $P(\hat{\pi})$ is given by the following:

 $P(\hat{\pi}) : \max_{\pi} \operatorname{maximize} U_{j}(\hat{\pi}), \text{ with } U_{j}(\hat{\pi})$ $= \sum_{j \in J} \sum_{i \in I(j)} \sum_{k \in K} \frac{g(|I(j)|)}{|I(j)|} \left(\log(\pi_{jk}) + \log(G_{ijk}) - \log\left(N_{0} + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}\right) \right) \right)$ $= \sum_{j \in J} \sum_{i \in I(j)} \sum_{k \in K} \frac{g(|I(j)|)}{|I(j)|} \left(\hat{\pi}_{jk} + \widehat{G}_{ijk} - \log\left(N_{0} + \sum_{j' \neq j} \exp(\log(\pi_{j'k} G_{ij'k}))\right) \right) \right)$ $= \sum_{j \in J} \sum_{i \in I(j)} \sum_{k \in K} \frac{g(|I(j)|)}{|I(j)|} \left(\hat{\pi}_{jk} + \widehat{G}_{ijk} - \log\left(\exp(\widehat{N}_{0}) + \sum_{j' \neq j} \exp(\widehat{\pi}_{j'k} + \widehat{G}_{ij'k})\right) \right).$ (5a)

Subject to
$$\log\left(\sum_{k\in K} \exp(\hat{\pi}_{jk})\right) - \log\left(p_j^{max}\right) \le 0, \forall j \in J, k \in K.$$
 (5b)

$$-\hat{\pi}_{jk} + \log\left(p_j^{min}\right) \le 0, \forall j \in J, k \in K.$$
(5c)

Proposition 3.1 The resulting optimization problem $P(\hat{\pi})$ is convex and hence can be efficiently solved for global optimality even with a large number of users.

Proof The first term of the objective is a linear function, thus concave (and convex). The second term contains log-sum-exp expression which is convex. The opposite of the sum of convex functions being concave, this completes the proof of the concavity of the objective function. As for the new constraints: constraints (5b) are convex by virtue of the properties of the log-sum-exp functions and (5c) are linear function and hence convex.

4 Distributed Power Control Approach

Although optimal, a central power allocation is complex and necessitates having recourse to a central control that harvest signaling information from eNBs to allocate power optimally. We turn here to distributed schemes to diminish complexity at the cost of slow convergence time and lower performance.

Here, we propose a cooperative algorithm that makes profit from the X2 interface between neighboring eNBs in LTE, Any local optimum π^* of the centralized convex problem (5a, b, c) must satisfy the KKT conditions, i.e. there exist unique Lagrange multipliers $\forall j \in J$ such that: Any local optimum π^* of the centralized convex problem (5a, b, c) must satisfy the Karush-Kuhn-Tucker (KKT) conditions, i.e. there exist unique Lagrange multipliers $\forall j \in J$ such that:

$$\frac{\partial U_j(\hat{\pi})}{\partial \hat{\pi}_{jk}} + \sum_{l \neq j} \frac{\partial U_l(\hat{\pi})}{\partial \hat{\pi}_{jk}} = \mu_j - \lambda_j^k; \quad \forall k \in K.$$
 (6a)

$$\mu_{j} \cdot \left(\log(P_{j}^{max}) - \log\left(\sum_{k \in K} \exp\left(\hat{\pi}_{jk}\right)\right) \right) = 0.$$
 (6b)

$$\lambda_j^k \cdot \left(\hat{\pi}_{jk} - \log(p_j^{min})\right) = 0; \quad \forall k \in K.$$
(6c)

$$\mu_j \ge 0 \text{ and } \lambda_j^k \ge 0; \quad \forall k \in K.$$
 (6d)

We come back to the solution space in π instead of $\hat{\pi}$. In particular, we have what follows:

$$\frac{\partial U_j(\pi)}{\partial \pi_{jk}} = \frac{\partial \hat{\pi}_{jk}}{\partial \pi_{jk}} \frac{\partial U_j(\hat{\pi})}{\partial \hat{\pi}_{jk}} = \frac{1}{\pi_{jk}} \frac{\partial U_j(\hat{\pi})}{\partial \hat{\pi}_{jk}}$$

Accordingly, we obtain the following set of equations:

$$\pi_{jk} \cdot \left(\frac{\partial U_j(\pi)}{\partial \pi_{jk}} + \sum_{l \neq j} \frac{\partial U_{lk}(\pi)}{\partial \pi_{jk}} \right) = \mu_j - \lambda_j^k; \forall k \in K.$$
(7a)

$$\mu_j \cdot \left(P_j^{max} - \sum_{k \in K} \pi_{jk} \right) = 0.$$
(7b)

$$\lambda_j^k \cdot \left(\pi_{jk} - p_j^{min}\right) = 0; \forall k \in K.$$
(7c)

$$\mu_j \ge 0 \quad and \quad \lambda_j^k \ge 0; \quad \forall k \in K.$$
 (7d)

Using the KKT conditions, we give a decomposition of the original problem into |J| subproblems. Following [2] we define the interference impact I_{ijk} for user *i* associated to BS *j* on RB *k* such as:

$$I_{ijk}(\pi_{-j}) = \sum_{l \neq j} \pi_{lk} G_{ilk} + N_0; \quad \forall i \in I(j).$$
(8)

Further, we define the derivative of $U_{ijk} = \frac{g(|I(j)|)}{|I(j)|} \log \left(\frac{\pi_{jk} G_{ijk}}{N_0 + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}} \right)$ relative to the interference impact as follows:

$$\frac{\partial U_{ijk}}{\partial I_{ijk}} = \frac{g(|I(j)|)}{|I(j)|} \frac{-1}{I_{ijk}}.$$

Using (8), condition (7a) can be re-written as:

$$\pi_{jk}\left(\frac{\partial U_{jk}}{\partial \pi_{jk}} - \sum_{l \neq j} \sum_{k \in K} \sum_{i \in I(l)} \frac{g(|I(j)|)}{|I(j)|} \frac{G_{ijk}}{I_{ijk}}\right) = \mu_j - \lambda_j^k; \quad \forall k \in K, \forall j \in J.$$
(9)

Given fixed interference and fixing the power profile of any eNB except eNB *j*, it can be seen that (9) and conditions (7b–d) are the KKT conditions of the following optimization sub-problems $\forall j \in J$:

$$\begin{aligned} \max_{\pi_{j}} \max_{i \in I(j)} & V_{j}(\pi_{j}, \pi_{-j}) \\ &= \sum_{k \in K} \sum_{i \in I(j)} \frac{g(|I(j)|)}{|I(j)|} \log \left(\frac{\pi_{jk} G_{ijk}}{N_{0} + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}} \right) - \sum_{k \in K} \pi_{jk} \alpha_{jk}. \end{aligned}$$
(10a)
Subject to :
$$\sum_{k \in K} \pi_{jk} \leq p_{j}^{max}, p_{j}^{min} \leq \pi_{jk}; \forall k \in K.$$
(10b)

where α_{jk} is the interference impact on RB k of eNB j on other eNBs, and given by:

$$\alpha_{jk} = \sum_{\substack{l \in J \\ l \neq j}} \sum_{\substack{i \in I(l) \\ |I(j)|}} \frac{g(|I(j)|)}{|I(j)|} \frac{G_{ijk}}{\left(\sum_{\substack{j' \in J \\ j' \neq l}} \pi_{j'k} G_{ij'k} + N_0\right)}.$$
(11)

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However, we choose to replace α_{jk} by $\bar{\alpha}_{jk} = \frac{\alpha_{jk}}{|J|}$, which is the mean interference impact on RB *k* inflicted by eNB *j* on other eNBs. Hence, we formulate a new non-cooperative game $G' = \langle J, S, \overline{V} \rangle$, where:

$$\overline{V}_{j}(\pi_{j},\pi_{-j}) = \sum_{k \in K} \left(\sum_{i \in I(j)} U_{ijk} - \bar{\alpha}_{jk} \pi_{jk} \right); \quad \forall j \in J.$$
(12)

The first term of the new utility function $\sum_{i \in I(j)} U_{ijk}$ is a non-decreasing function in π_{jk} while the second term $-\bar{\alpha}_{jk}\pi_{jk}$ is decreasing in π_{jk} , which permits to strike a good balance between spectral efficiency and energy efficiency. Hence, the higher is the mean interference harm inflected on neighboring eNBs on a given RB *k*, the lower will be the chosen power amount π_{jk} .

For every *j*, \overline{V}_j is concave w.r.t. π_j and continuous w.r.t. π_l , $l \neq j$. Hence, a Nash Equilibrium (NE) exists [3]. Furthermore, the game at hand is super-modular. In fact, the strategy space S_j is obviously a compact convex set of \mathbb{R}^k , while the objective function of any eNB *j* is super-modular [4]:

$$\begin{aligned} \frac{\partial \overline{V}_{jk}}{\partial \pi_{lk} \partial \pi_{jk}} \\ &= \sum_{\substack{s \in J \\ s \neq \{j,l\}}} \sum_{i \in I(s)} \frac{g(|I(s)|)}{|J||I(s)|} \frac{G_{ijk}G_{ilk}}{\left(\sum_{j' \in J, j' \neq s} \pi_{j'k}G_{ij'k} + N_0\right)^2} \left(1 - \frac{G_{ijk}\pi_{jk}}{\left(\sum_{j' \in J, j' \neq s} \pi_{j'k}G_{ij'k} + N_0\right)}\right) \ge 0. \end{aligned}$$

 $\forall l \in J - \{j\}$ and $\forall k \in K$, as we can fairly assume with at least 6 neighboring eNBs for any eNB *s* that $\frac{G_{ijk}\pi_{jk}}{\left(\sum_{j' \in J, j' \neq s} \pi_{j'k}G_{ij'k} + N_0\right)} < 1.$

As we proved that we are in presence of a super-modular game, we know that a Best response algorithm enables attaining the NEs. The main idea behind this algorithm is for each eNB j to iteratively solve the optimization problem in (10a, b) given the current interference impact and power profile of the other eNBs and then to recalculate the corresponding interference impact until convergence. Formally, we summarize this as follows:

- 1. Each eNB *j* chooses an initial power profile π_i satisfying the power constraint.
- 2. Using (11), each eNB *j* calculates the mean interference price vector $\bar{\alpha}_j$ given the current power profile and announces it to other eNBs.
- 3. At each time *t*, one eNB *j* is randomly selected to maximize its payoff function $\overline{V}_j(\pi_j, \pi_{-j})$ and update its power profile, given the other eNBs power profiles π_{-j} and price vectors, i.e., $\pi_j(t+1) = \arg \max_{\pi_i \in S_j} \overline{V}_j(\pi_j, \pi_{-j}(t))$.

4.1 The Power Expression at Equilibrium

We begin by solving the unconstrained convex optimization problem $\max_{\pi_j} \overline{V_j}(\pi_j, \pi_{-j})$. Then, to obey the bounding constraints on power levels, any eNBs *j* must do locally a projection step in order to get back to the feasible region defined by S_j. The optimal values of the unconstrained problem are either on the boundaries of the strategy space or resulting from the following derivation $\forall j \in J, \forall k \in K$:

$$\frac{\partial \overline{V}_{j}(\pi_{j},\pi_{-j})}{\partial \pi_{ik}} = 0 \Rightarrow$$
(13a)

$$\pi_{jk}^{2} \cdot \left(\sum_{l \neq j} \sum_{i \in I(l)} G_{ijk}^{2} C_{l} B_{jl} \right) + \pi_{jk} \cdot \left(\sum_{l \neq j} \sum_{i \in I(l)} G_{ijk} A_{ik} C_{l} (2B_{jl} - 1) \right) + \sum_{j \neq j} \sum_{i \in I(l)} C_{l} A_{ik}^{2} = 0$$
(13b)

where

$$C_{l} = \frac{g(|I(j)|)}{|\mathsf{J}||I(j)|}, B_{jl} = \frac{g(|I(j)|)}{g(|I(j)|)} \text{ and } A_{ik} = \left(\sum_{\substack{j' \in J \\ j' \neq \{j, l\}}} \pi_{j'k} G_{ij'k} + N_{0}\right).$$

Consequently, π_{jk} is the solution of the second degree equation in (13b). After obtaining the various $s\pi_{jk}$, the projection algorithm 1 is run by every eNB *j* at each iteration as follows:

	Algorithm 1 Projection algorithm for eNB j		
1:	procedure POWERPROJECTION (π_i)		
2:	$S(K) \leftarrow SORTINDECREASINGORDER(K)$		
3:	for all $k \in s(K)$ do		
4:	if $\pi_{jk} < p_j^{min}$ then		
5:	$\pi^p_{jk} \leftarrow P^{min}_j$		
6:	end if		
7:	end for		
8:	if $\pi_j \not\in S_j$ then		
9:	$\rho(k) \leftarrow \pi_{jk} + \frac{1}{k} \times \left(P_j^{maz} - \sum_{i \in S(K), i \leq k} \pi_{ji} \right)$		
1/	$\forall k \in S(K) \text{ and } n_{jk} > r_j$		
10	$\rho \leftarrow \operatorname{argmax}_{k \in s(K)} \{ v(k) \}$		
11	$\lambda \leftarrow \frac{1}{\rho^*} \times \left(P_j^{max} - \sum_{i \in S(K), i \leq k} \pi_{ji} \right)$		
12	for all $k \in S(K)$ do		
13	: if $\pi_{jk} > p_j^{min}$ then		
14	$: \qquad \qquad \boldsymbol{\pi}_{j,k}^{p} \leftarrow \max\{\pi_{jk} - \lambda, p_{j}^{\min}\}$		
15	end if		
16	end for		
17	/: end if		
18	Return $\pi_j^p = \left(\pi_{j,k}^p, \forall k \in K\right)$		
19	end procedure		

5 Performance Evaluation

We consider a Bandwidth of 5 MHz with 25 RBs in a 9 hexagonal cells network, the number of UE ranging from 4 to 14 per eNB uniformly distributed in any cell. Further, we consider the following parameters listed in the 3GPP technical specifications TS 36.942 [5]: the mean antenna gain in urban zones is 12 dBi (900 MHz). Transmission power is 43 dBm (according to TS 36.814) which corresponds to 20 W (on the downlink). The eNBs have a frequency reuse of 1, with W = 180 kHz. As for noise, we consider the following parameters: user noise figure 7.0 dB, thermal noise -104.5 dBm which gives a receiver noise floor of pN = -97.5 dBm.

In this chapter, we conducted preliminary simulations in a Matlab simulator, where various scenarios were tested to assess the performances of the power control schemes.

For each approach, 25 simulations were run, where in each cell a predefined number of users is selected; users' positions were uniformly distributed in the cells. For each simulation instance, the same pool of RBs, users and pathloss matrix are given for both algorithms.

In Fig. 1, we can see the similarity of power economy efficiency between the centralized algorithm and the semi-distributed algorithm. Both power control schemes permit a considerable power economy in comparison with the Max Power policy, that uses full power P_{max}^{j} for each eNB, as we can see in Fig. 1 where the power economy percentage for all eNBs vary from 55 to 65% in comparison with the Max power policy, which is a sensible power economy.

In fact, the existence of the power cost $-\sum_{k \in K} \pi_{jk} \bar{\alpha}_{jk}$ in the utility function (12), diminishes the selfishness of eNBs that are tempted to transmit at full power on all RBs.

This power economy is obtained while maintaining good performances as we can see in Fig. 2 where the utility function in (3) is depicted as a function of the number of users for the centralized algorithm and the semi-distributed algorithms.

In Fig. 3, we report the mean convergence time per eNBs for the semi-distributed algorithm for various scenarios. We note that each eNBs attains in average the NE within 19–27 iterations. At each iteration, one eNB is randomly selected to maximize its payoff function given in (13). The iteration period coincides with one TTI (Transmit Time Interval), which equals 1 ms in LTE.

We noted during the extensive simulations conducted, that the power levels attain 90% of the values reached at convergence in less than 8 iterations. We represented in Fig. 4 the power distribution of 25 RBs for an eNB selected randomly and for which convergence time was equal to 22 iterations. Low convergence time in conjunction with high performances is an undeniable asset for our semi-distributed schemes.

The Decentralized algorithms can adapt to fast changes of network state though it is difficult to avoid converging to local optimum. It turns out that even though the distributed game results are sub-optimal, the low degree of system complexity and



Fig. 1 Percentage of power economy as a function of the number of users for centralized and semi-distributed versus Max power algorithms



Fig. 2 Utility function as function of the number of users for centralized and semi-distributed



Fig. 3 Total convergence time by eNBs as function of the number of users for semi-distributed algorithm



Fig. 4 Power distribution by RBs before reaching convergence for semi-distributed algorithm

the inherent adaptability make the decentralized approach promising especially for dynamic scenarios. The fast convergence time, the near optimal results and the lower complexity degree of the semi-distributed approach makes it a very attractive solution.

6 Conclusion

In this chapter, the power levels are astutely set as part of the LTE Inter-cell Interference coordination process in smart cities. We proposed a non-cooperative game and a best response algorithm to reach the NEs of the portrayed game. This semi-distributed algorithm astutely and efficiently set the power levels with relatively low convergence time. Numerical simulations assessed the good performances of the proposed approach in comparison with the optimal centralized approach. More importantly, considerable power economy and signaling optimization can be realized.

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