Joint Scheduling and Power Control in Multi-Cell Networks for Inter-Cell Interference Coordination

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***Abstract*—The focus of this paper is targeted towards multi-cell dense LTE and LTE-Advanced networks, which are composed of multiple evolved Node B (eNodeB) co-existing in the same operating area and sharing the available radio resources. In such scenarios, momentous emphasis is given towards the techniques that take Inter-Cell Interference (ICI) into account while allocating the scarce radio resources. In this context, we propose solutions for the problem of joint power control and scheduling in the framework of Inter-Cell Interference Coordination (ICIC) in the downlink of LTE OFDMA-based multi-cell systems. Two approaches are adopted to allocate system resources in order to achieve high performance: a centralized approach based on convex optimization and a semi-distributed approach based on non-cooperative game theory. The centralized approach needs a central controller to optimally allocate resources like in LTE CoMP (Coordinated Multipoint). In the semi-distributed approach, eNodeBs coordinate among each other for efficient resource allocation based on local knowledge conveyed by the X2 interface. It turns out that despite the lower complexity of the semi-distributed approach and its inherent adaptability, there is only a slight discrepancy of results among both approaches, which makes the distributed approach much more promising, in particular as a procedure of SON (Self Organized Network).**

Keywords—ICIC, LTE, Convex Optimization, Game Theory, Geometric Programming, and OFDMA.

I. INTRODUCTION

Owing to its numerous merits, Orthogonal Frequency Division Multiple Access (OFDMA) is widely accepted as the access scheme for the downlink of LTE [1] and LTE-Advanced (LTE-A) [2] networks. However, the Inter-Cell Interference (ICI), especially when the frequency resources are universally reused in each cell for high Spectral Efficiency (SE), can be very harmful. To address this challenging problem, Inter-Cell Interference Coordination (ICIC) is commonly identified as a key radio resource management mechanism. One of the efficient ICIC approaches is joint power control and scheduling procedures. Unfortunately, the complexity of such approach is exponential as a function of network size. Therefore, the successful development of any resource allocation scheme for OFDMA-based networks certainly relies upon how we can effectively overcome such cumbersome problem. Moreover, joint scheduling and power allocation schemes [3] that guarantee acceptable performances per user are among the challenges of 5G networks envisioned to achieve better user experience and higher operator profits.

In this paper, we formulate the joint scheduling and power allocation problem for multi-cell OFDMA-based networks. We prove that the original problem is separable into two independent optimization problems: a scheduling problem and a power allocation problem. Our objective is to strike a good balance between fairness and efficiency through maximizing the achievable Signal-to-Interference and Noise Ratio (SINR). In particular, the power allocation problem is initially solved in a centralized way; the resulting optimization problem is rendered convex through geometric transformation. Then, a semi-distributed version is presented and casted as a non-cooperative game where each eNodeB tries to optimize locally its own performances and communicates its power level to its neighbors until convergence.

The paper is organized as follows. Section II describes the related work. Section III presents the network model and introduces the network utility function. Section IV presents the power level selection scheme as a non-cooperative game for the semi-distributed approach. Section V presents the simulations results. Section VI concludes the paper with a summary of the findings works and section VII provides proofs of our propositions.

II. RELATED WORK

Joint power control and scheduling algorithms have been studied extensively in the literature. In the following, we investigate some of the most important works related to our approach.

In [4], two power control algorithms are proposed to automatically create Soft Fractional frequency Reuse (SFR) patterns in OFDMA-based systems. The goal of the proposed algorithms is to adjust the transmit powers of the different sub-channels by systematically pursuing maximization of the overall network utility. The first algorithm is semi-distributed as relies on gradient information exchanged periodically by neighboring cells, whereas the second one is fully distributed relying on a non-trivial heuristic. The work in [5] builds upon the work in [4] by extending the proposed algorithms for multi antenna OFDM systems with space division multiple access. In [6], the power level selection process of resource blocks (RB) is apprehended as a non-cooperative sub-modular game. In [7], the joint allocation of RBs and transmit powers is investigated for the downlink transmission of OFDMA-based femtocells, modeled by an exact potential game. In [8], a joint subchannel and binary power allocation algorithm is proposed, where only one transmitter is allowed to send signals on each subchannel. In [9], various iterative schemes are proposed to centrally solve the problem of joint power allocation and scheduling in a coordinated OFDMA multi-cell network. The work in [10] proposes several joint subchannel and power allocation schemes for OFDMA femtocells based on Lagrangian dual relaxation. Finally, in [11], an iterative approach is devised in which OFDM subchannels and power levels of base stations are alternatively assigned and optimized at every step.

III. Network Model

We consider a cellular network comprising a set of eNodeBs denoted by *J*. We focus on the downlink in this paper. The time and frequency radio resources are grouped into time-frequency Resource Blocks (RBs). An RB is the smallest radio resource unit that can be scheduled to a mobile user. Each RB consists of N_s OFDM symbols in the time dimension and N_f sub-carriers in the frequency dimension (in LTE N_s=7 and N_f=12). The set of RBs is denoted by *K*, and the set of users is denoted by *I*. Both eNodeBs and mobile users have a single antenna each. In the following, we make the following assumptions:

1. We consider a fixed cell assignment and denote by *I(j)* the set of users associated to eNodeB $j \in J$. Each user typically compares the received signal power from each eNodeB and chooses to connect with the best received eNodeB.
2. We consider permanent downlink traffic where each eNodeB has persistent traffic towards its users. We also assume that all RBs are assigned on the downlink at each scheduling epoch.

Symbols, variables and parameters used within this paper are defined in Table 1

TABLE I. Symbols, variables and parameters in the document

$\boldsymbol{J}$	Set of eNodeBs	$\boldsymbol{I(j)}$	Set of users associated to eNodeB *j*
$\boldsymbol{I}$	Total set of users	$\boldsymbol{G_{ijk}}$	Channel power gain (user *i* on RB *k* on eNodeB *j*)
$\boldsymbol{N_0}$	Noise power	$\boldsymbol{\rho_{ijk}}$	SINR of user *i* associated eNodeB *j* served on RB *k*
$\boldsymbol{K}$	Set of Resource blocks	$\boldsymbol{\alpha_{jk}}$	Interference impact of eNodeB i among other eNodeBs
$\boldsymbol{\pi_{jk}}$	Transmit power of eNodeB *j* on RB *k*	$\boldsymbol{\theta_{ik}}$	Percentage of time user *i* is associated with RB *k*

A. Power Consumption Model

The power consumption of eNodeB $j \in J$ is modeled as a linear function [12] of the average transmit power per site as below:

$$p_j = \zeta_j^1 \pi_j + p_j^0. \quad (1)$$

where p_j and π_j denote the average consumed power by eNodeB *j* and its transmit power, respectively. The coefficient ζ_j^1 accounts for the power consumption that scales with the transmit power due to radio frequency amplifier and feeder losses while p_j^0 models the power consumed independently of the transmit power due to signal processing and site cooling.

The transmit power of each eNodeB is allocated to resource blocks serving the users in the network. The total transmit power of eNodeB *j* is the sum of the transmit power on each RB $k \in K$:

$$\pi_j = \sum_{k \in K} \pi_{jk}. \quad (2)$$

where π_{jk} is the transmit power of eNodeB *j* on RB *k*, hence, the total power consumed by any eNodeB *j* is given by:

$$P_j = \zeta_j^1 \sum_{k \in K} \pi_{jk} + p_j^0. \quad (3)$$

B. SINR Model

Given user *i* served by eNodeB *j* ($i \in I(j)$), the signal-to-interference-plus-noise-ratio (SINR) of this user when served on RB *k* is given by:

$$\rho_{ijk} = \frac{\pi_{jk} G_{ijk}}{N_0 + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}} \quad (4)$$

where G_{ijk} is the path gain of user *i* towards eNodeB *j* on resource block *k* (computed as an average over the sub-carriers in the resource block), and N_0 is the noise power, which is, without loss of generality, assumed to be the same for all users on all resource blocks.

C. Utility function Model

Let θ_{ik} denotes the percentage of time user *i* is scheduled on resource block *k*. We consider the below global utility function for the system:

$$U(\theta, \pi) = \sum_{j \in J} \sum_{i \in I(j)} \sum_{k \in K} \log(\theta_{ik} \rho_{ijk}) \quad (5)$$

$$with\ 0 \leq \theta_{ik} \leq 1.$$

The above utility function insures that the deviation between the highest and lowest throughput over all users is as small as possible. This will provide fairness in the system using a mathematically tractable optimization problem.

The utility function presented in (5) is linearly separable into two different optimization problems: a scheduling problem $U(\theta)$, that computes the percentage of time user *i* is served on each RB *k* by eNodeB *j*, and a power allocation problem$U(\pi)$:

$$U(\theta, \pi) = U(\theta) + U(\pi).$$

where $U(\theta)$ and $U(\pi)$ are given by what follows:

$$U(\theta) = \sum_{j \in J} \sum_{i \in I(j)} \sum_{k \in K} \log(\theta_{ik}). \quad (6a)$$

$$U(\pi) = \sum_{j\in J}\sum_{i\in I(j)}\sum_{k\in K}\log\left(\frac{\pi_{jk}G_{ijk}}{N_0+\sum_{j'\neq j}\pi_{j'k}G_{ij'k}}\right) \quad (6b)$$

$$with\ 0 \le \theta_{ik} \le 1.$$

D. The Scheduling Problem

Based on (6a), the utility function of the scheduling problem is independent of j and hence can be solved locally by each eNodeB j:

$$U(\theta) = \sum_{j\in J}\sum_{i\in I(j)}\sum_{k\in K}\log(\theta_{ik}) = |J|.\sum_{i\in I(j)}\sum_{k\in K}\log(\theta_{ik}).$$

Accordingly, the scheduling problem per cell can be written as the following optimization problem $(\widehat{\mathrm{P}}_{\mathrm{sched}})$:

$$\underset{\theta}{\text{maximise}} \quad U_j(\theta) = \sum_{i\in I(j)}\sum_{k\in K}\log(\theta_{ik}). \quad (7a)$$

$$\text{Subject to} \quad \sum_{i\in I(j)}\theta_{ik} \le 1, \forall k \in K. \quad (7b)$$

$$\sum_{k\in K}\theta_{ik} \le 1, \forall i \in I(j). \quad (7c)$$

$$0 \le \theta_{ik} \le 1, \forall i \in I(j), \forall k \in K. \quad (7d)$$

Proposition 3.1 The optimal solution of the per scheduling problem is given by:

$$\forall i \in I(j), \forall k \in K, \theta^*_{i,k} = \begin{Bmatrix} \frac{1}{|I(j)|}, if\ |K| \le |I(j)| \\ \frac{1}{|K|}, otherwise \end{Bmatrix}. \quad (8)$$

The proof is provided in the appendix.

E. The Centralized Power Control Problem

Based on (6b), the power control problem can be written as the following optimization problem $P(\pi)$:

$$\underset{\pi}{\text{maximise}} \quad U(\pi) = \sum_{j\in J}\sum_{i\in I(j)}\sum_{k\in K}\log\left(\frac{\pi_{jk}G_{ijk}}{N_0+\sum_{j'\neq j}\pi_{j'k}G_{ij'k}}\right). \quad (9a)$$

$$subject\ to \quad \sum_{k\in K}\pi_{jk} \le p_j^{max}, \forall j \in J. \quad (9b)$$

$$\pi_{jk} \ge p_j^{min}, \forall j \in J, \forall k \in K. \quad (9c)$$

Problem (9) is a non-linear and non-convex optimization problem. However, it can be transformed into a convex optimization problem in the form of geometric programming by performing a variable change $\hat{\pi}_{jk} = \log(\pi_{jk})$ and defining $\hat{N}_0 = \log(N_0)$ and $\hat{G}_{ijk} = \log(G_{ijk})$.

The resulting optimization problem deemed $P(\hat{\pi})$ is given by what follows:

$$\text{maximise}_{\hat{\pi}} \quad U_j(\hat{\pi}) = \sum_{j\in J}\sum_{i\in I(j)}\sum_{k\in K}(\hat{\pi}_{jk}+\hat{G}_{ijk}) + \sum_{j\in J}\sum_{i\in I(j)}\sum_{k\in K}\left(-log\left(\exp(\hat{N}_0)+\sum_{j'\neq j}\exp(\hat{\pi}_{j'k}+\hat{G}_{ij'k})\right)\right). \quad (10a)$$

$$\text{Subject to:} \quad \log\left(\sum_{k\in K}\exp(\hat{\pi}_{jk})\right) - log(p_j^{max}) \le 0. \quad (10b)$$

$$-\hat{\pi}_{jk} + \log(p_j^{min}) < 0, \forall j \in J, \forall k \in K. \quad (10c)$$

Proposition 3.2 The resulting optimization problem $P(\hat{\pi})$ is convex and hence can be very efficiently solved for global optimality even with a large number of users.

The proof is provided in the appendix.

IV. DISTRIBUTED POWER CONTROL

We have solved the problem $P(\pi)$,which is a convex problem, in a centralized fashion. In general, central entities performing the task of interference coordination with global knowledge should be avoided because they easily become bottlenecks in the network. Therefore, our work strives to obtain a semi-decentralized scheme that exploits the existence of X2 interface between neighboring eNodeBs in LTE.

Any optimum π^* of the centralized convex problem (10) must satisfy the Karush Kuhn Tucker (KKT) conditions, i.e. there exist unique Lagrange multipliers $\forall j \in J$ such that:

$$\frac{\partial U_j(\hat{\pi})}{\partial\hat{\pi}_{jk}} + \sum_{l\neq j}\frac{\partial U_{lk}(\hat{\pi})}{\partial\hat{\pi}_{jk}} = \mu_j - \lambda_j^k;\ \forall k \in K. \quad (11a)$$

$$\mu_j.\left(\log(P_j^{max}) - \log\left(\sum_{k\in K}\exp(\hat{\pi}_{jk})\right)\right) = 0. \quad (11b)$$

$$\lambda_j^k.\left(\hat{\pi}_{jk} - \log(p_j^{min})\right) = 0; \forall k \in K. \quad (11c)$$

$$\mu_j \ge 0\ and\ \lambda_j^k \ge 0\ \forall k \in K. \quad (11d)$$

where $U_{jk} = \sum_{i\in I(j)} log\left(\frac{\pi_{jk}G_{ijk}}{N_0+\sum_{j'\neq j}\pi_{j'k}G_{ij'k}}\right)$.

We come back to the solution space in π instead of $\hat{\pi}$. In particular, we have what follows:

$$\frac{\partial U_{jk}(\pi)}{\partial\pi_{jk}} = \frac{\partial\hat{\pi}_{jk}}{\partial\pi_{jk}}\frac{\partial U_{jk}(\hat{\pi})}{\partial\hat{\pi}_{jk}} = \frac{1}{\pi_{jk}}\frac{\partial U_{jk}(\hat{\pi})}{\partial\hat{\pi}_{jk}}.$$

Accordingly, we obtain the following set of equations:

$$\pi_{jk}.\left(\frac{\partial U_{jk}(\pi)}{\partial\pi_{jk}} + \sum_{l\neq j}\frac{\partial U_{lk}(\pi)}{\partial\pi_{jk}}\right) = \mu_j - \lambda_j^k;\ \forall k \in K. \quad (12a)$$

$$\mu_j.\left(P_j^{max} - \sum_{k\in K}\pi_{jk}\right) = 0. \quad (12b)$$

$$\lambda_j^k.\left(\pi_{jk} - p_j^{min}\right) = 0; \forall k \in K. \quad (12c)$$

$$\mu_j \ge 0\ and\ \lambda_j^k \ge 0\ \forall k \in K. \quad (12d)$$

Using the KKT conditions, we give a decomposition of the original problem into $|J|$ subproblems. Following [13], we define the interference impact I_{ijk} for user i associated to eNodeB j on RB k such as:

$$I_{ijk}(\pi_{-j}) = \sum_{l\neq j}\pi_{lk}G_{ilk} + N_0, \forall i \in I(j). \quad (13)$$

Further, we define the derivative relative to the interference impact of $U_{ijk} = log\left(\frac{\pi_{jk}G_{ijk}}{N_0+\sum_{j'\neq j}\pi_{j'k}G_{ij'k}}\right)$ as follows:

$$\frac{\partial U_{ijk}}{\partial I_{ijk}} = \frac{-1}{I_{ijk}}.$$

using (15), condition (14a) can be written as:

$$\pi_{jk}.\frac{\partial U_{jk}}{\partial \pi_{jk}} - \sum_{l\neq j}\sum_{i\in I(l)} \pi_{jk}.\frac{G_{ijk}}{I_{ijk}} = \mu_j - \lambda_j^k. \quad (14)$$

$$\forall k \in K, \forall j \in J$$

Given fixed interference and fixing the power profile of any eNodeB except eNodeB j, it can be seen that (14) and conditions (12b)-(12d) are the KKT conditions of the following optimization sub-problems $\forall j \in J$:

$$\underset{\pi_j}{\text{maximize}}\, V_j(\pi_j, \pi_{-j})$$

$$= \sum_{k\in K} \pi_{jk} U_{jk} - \sum_{k\in K}\int U_{jk} d\pi_{jk} - \sum_{k\in K} \pi_{jk}^2 \alpha_{jk}.$$

$$= \sum_{k\in K} \pi_{jk}\left(|I(j)| - \pi_{jk}\alpha_{jk}\right). \quad (15)$$

$$subject\ to \quad \sum_{k\in K} \pi_{jk} \leq p_j^{max}, p_j^{min} \leq \pi_{jk}, \forall k \in K.$$

where α_{jk} is the interference impact of eNodeB j on other eNodeBs $l \neq j$, and given by:

$$\alpha_{jk} = \frac{1}{2}.\sum_{\substack{l\in J\\ l\neq j}}\sum_{i\in I(l)} \frac{G_{ijk}}{\left(\sum_{\substack{j'\in J\\ j'\neq l}} \pi_{j'k}G_{ij'k} + N_0\right)}. \quad (16)$$

Resorting to non-cooperative game theory is quite suitable to model the way eNodeBs compete in a distributed manner for limited resources. Devising an optimal power level selection scheme depends on the existence of Nash equilibriums for the present game which will be explored in what follows.

A. Non-Cooperative Game for power allocation

Non-cooperative game theory models the interactions between players competing for a common resource. Hence, it is well adapted to power allocation modeling. Here, eNodeBs are the decision makers or players of the game. We define a multi-player game G between the |J| eNodeBs which are assumed to make their decisions without knowing the decisions of each other.

The formulation of this non-cooperative game $G=\langle N, S, V\rangle$ can be described as follows:

- A finite set of eNodeBs J=(1,...,|J|) and a finite set of RBs K=(1,...,|K|).
- For each eNodeB j, the space of pure strategies S_j is as follows:

$$S_j = \begin{Bmatrix} \pi_j \in R^{|K|} such\ as\ \pi_{jk} \geq p_j^{min} \\ and \sum_{k\in K}\pi_{jk} \leq p_j^{max}, \forall k \in K \end{Bmatrix}.$$

- An action of an eNodeB j is the amount of power $\pi_{j,k}$ sent on RB k. The strategy chosen by eNodeB j is then $\pi_j = (\pi_{j,1}, \dots, \pi_{j,k})$. A strategy profile $\pi = (\pi_1, \dots, \pi_{|J|})$ specifies the strategies of all players and $S = S_1 \times \dots \times S_{|J|}$ is the set of all strategies.
- A set of utility functions V=($V_1(\pi)$, $V_2(\pi)$,..., $V_{|J|}(\pi)$) that quantify players' utility for a given strategy profile π where the utility function V_j of a given eNodeB j is as follows:

$$V_j(\pi_j, \pi_{-j}) = \sum_{k\in K}\left(\sum_{i\in I(j)}|I|\pi_{jk} - \alpha_{jk}\pi_{jk}^2\right).$$

Note that, the first term of the new utility function $V_j(\pi_j, \pi_{-j})$ is a non-decreasing function in π_{jk} while the second term $-\alpha_{jk}\pi_{jk}^2$ is decreasing in π_{jk} which permits to strike a good balance between spectral efficiency and energy efficiency. Hence, the higher is the interference harm inflected by eNodeB j on neighboring eNodeBs on a given RB k, the lower will be the chosen power amount π_{jk}. This will restrain selfish eNodeBs from transmitting at the maximum allowable power per RB.

Furthermore, as $V_j(\pi_j, \pi_{-j})$ is convex w.r.t. π_j and continuous w.r.t. π_y, $y\neq j$, Nash equilibriums exist according to [14]. We turn to S-modularity theory [15] to obtain an algorithm that can attain the Nash equilibriums of the game $G=\langle N, S, V\rangle$.

B. The SupermodularPower Control Game

S-modularity was introduced into the game theory literature by [15] in 1979. S-modular games are of particular interest since they have Nash equilibriums, and there exists an upper and a lower bound on Nash strategies of each user [16]. More importantly, these equilibriums can be attained by using a greedy best response type algorithm.

Definition 4.1: consider a game $G=\langle N,S,V\rangle$ with strategy spaces $S_j \subset \mathbb{R}^K$ for all $j \in J$ and $k \in K$, G is super-modular if for each j, S_j is a sublattice[1] of $\mathbb{R}^K$, and $V_j(\pi_j, \pi_{-j})$ is a super-modular function.

Since S_j is a convex and compact subset of $\mathbb{R}^K$, it is a sublattice of $\mathbb{R}^K$.

Definition 4.2: If the utility function $V_j(\pi_j, \pi_{-j})$ is twice differentiable, it is super-modular if: $\frac{\partial V_j(\pi_j,\pi_{-j})}{\partial \pi_{j,k}\partial \pi_{y,k}} \geq 0$ for all $y\neq j \in J$, for all $k \in K$ and for any feasible strategy. We need only to check whether the utility function is super-modular for any eNodeB j and any RB which is straightforward as the following derivative is positive:

$$\frac{\partial V_j(\pi_j, \pi_{-j})}{\partial \pi_{j,k}\partial \pi_{y,k}} = \sum_{\substack{l\in J\\ l\neq j}}\sum_{i\in I(l)} \frac{G_{ijk}\pi_{jk}G_{iyk}}{\left(\sum_{\substack{j'\in J\\ j'\neq l}} \pi_{j'k}G_{ij'k} + N_0\right)^2} \geq 0 \quad (17)$$

Therefore, our game is indeed super-modular.

[1] A is sublattice of $\mathbb{R}^m$ if a and $a' \in A$ imply $a \wedge a' \in A$ and $a \vee a' \in A$

C. Attaining the Nash Equilibrium

The Best response strategy of player j is the one that maximizes its utility given other players strategies. A best power response scheme consists of a sequence of rounds; each eNodeB j chooses the best response to the other eNodeBs strategies in the previous round. In some games, the sequence of strategies generated by best power response converges to a NE, regardless of the players' initial strategies. S-modular games are part of those games.

Hence, the main idea behind the best power response is for each eNodeB j to iteratively solve the optimization problem in (15) given the current interference impact and power profile of the other eNodeBs and then to recalculate the corresponding interference impact until convergence. Formally, we summarize this as follows:

1. Each eNodeB j chooses an initial power profile π_j satisfying the power constraint.
2. Using (16), each eNodeB j calculates the interference price vector α_j given the current power profile and announces it to other eNodeBs.
3. At each time t, one eNodeB j is randomly selected to maximize its payoff function $V_j(\pi_j, \pi_{-j})$ and update its power profile, given the other eNodeBs power profiles π_{-j} and price vectors, i.e.:

$$\pi_j(t+1) = arg\max_{\pi_j} V_j(\pi_j, \pi_{-j}(t)) \quad (18)$$

Finding the best response strategy comes down to obtaining the optimal solution of (15). To compute the optimal power solution π_j for any eNodeB j, we have recourse to the Lagrangian method. Accordingly, we write the Lagrangian of problem (15) as follows:

$$L(\pi_j, \beta, \gamma_1, \dots, \gamma_k) = \sum_{k\in K} \pi_{jk}\left(|I(j)| - \pi_{jk}\alpha_{jk}\right) + \beta\left(P_j^{max} - \sum_{k\in K}\pi_{jk}\right) + \sum_{k\in K}\gamma_k\left(\pi_{jk} - p_j^{min}\right). \quad (19)$$

where $\beta \geq 0$ and $\gamma_k \geq 0, \forall k \in K$ are the Lagrangian multipliers.

The dual problem in (19) is as follows:

$$\min_{\substack{\beta\geq 0\\ \gamma_1\dots\gamma_k\geq 0}} g(\beta, \gamma_1 \dots \gamma_k) = \min_{\substack{\beta\geq 0\\ \gamma_1\dots\gamma_k\geq 0}} \max_{\pi_j} L(\pi_j, \beta, \gamma_1 \dots \gamma_k) \quad (20)$$

As $L(\pi_j, \beta, \gamma_1, \dots, \gamma_k)$ is a standard concave function, each eNodeB j derives the optimal power levels by seeking zero points of the derivatives of $L(\pi_j, \beta, \gamma_1, \dots, \gamma_k)$. The power-allocation equations are:

$$|I(j)| - 2\alpha_{jk}\pi_{jk} = \beta + \gamma_k. \quad (21)$$

Accordingly, we obtain:

$$\pi_{j,k}(t) = \frac{|I(j)| - \beta(t) - \gamma_k(t)}{2.\alpha_{jk}(t)}. \quad (22)$$

Finally, to obtain the required power levels, we use a gradient method to update the dual variables β and $\gamma_k, \forall k \in K$ since $g(\beta, \gamma_1, \dots, \gamma_k)$ is differentiable:

$$\frac{\partial g(\beta, \gamma_1, \dots, \gamma_k)}{\partial \beta} = P_j^{max} - \sum_{k\in K}\pi_{jk} \quad (23)$$

$$\frac{\partial g(\beta, \gamma_1, \dots, \gamma_k)}{\partial \gamma_k} = \pi_{jk} - p_j^{min}.$$

Hence, β *and* γ_k *variables are updated* $\forall k \in K$ *as follows:*

$$\beta(t) = \max\left(0, \beta(t-1) - \delta_t\left(P_j^{max} - \sum_{k\in K}\pi_{jk}(t-1)\right)\right)$$
$$\gamma_k(t) = \max\left(0, \gamma_k(t-1) - \delta_t\left(\pi_{jk}(t-1) - p_j^{min}\right)\right) \quad (24)$$

where δ_t is a suitably small step size.

V. SIMULATION RESULTS

We consider a network with hexagonal cells, where each cell is surrounded by 6 others. The physical layer parameters are based on 3GPP technical specifications TS 36.942 [17]. These parameters and the simulation parameters are displayed in Table 2.

In this paper, we conducted preliminary simulations in a Matlab simulator, where various scenarios were tested to assess the performances of the two power control schemes.

TABLE II. PHYSICAL LAYER AND SIMULATION PARAMETERS

Channel bandwidth (MHz)	5	***Number of RBs***	25
Thermal noise (dBm)	-174	***Time subframe TTI (ms)***	1
Max power/eNodeB (dBm)	43	***Min Power/RB (dBm)***	15
Number user/eNodeB	8	***Number eNodeBs***	9
Antenna configuration	1-transmit, 1-receive SISO (Single Input Single Output)		

For each approach, 25 simulations were run where in each cell a predefined number of users is selected; users' positions were uniformly distributed uniformly in the cells . For each simulation instance, the same pool of RBs, users and pathloss matrix are given for both algorithms (Centralized and Semi-distributed).

A. Performance Evaluation

In Fig.1, we depict the histogram of the SINR for the centralized approach vs. the semi-distributed algorithm. As we can see, the SINR distribution is equivalent for both approaches for which more than 91% of the SINR is greater than 10 dB. More importantly, we see that both approaches have almost similar performances, which favor the semi-distributed approach owing to its lower complexity.

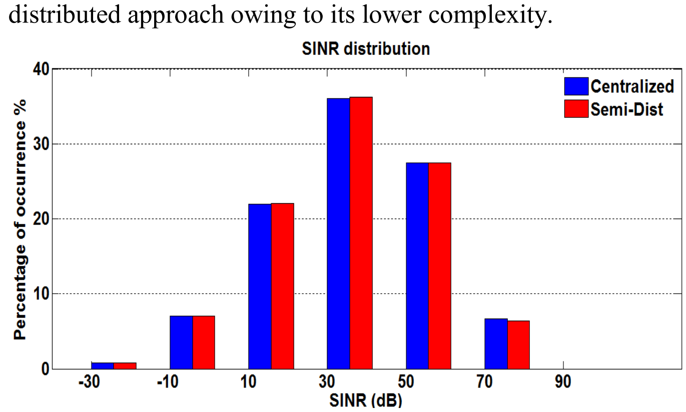

Fig 1. Percentage of SINR distribution occurences for centralized vs. semi-distributed algorithms

In Fig.2, we depict the histogram of the power ratio, defined as π_{jk}/P^{j}_{max}, for both approaches. For the semi-distributed strategy, we display the power distribution after convergence. Here, we see the discrepancy in the power distribution between both strategies. For the semi-distributed approach, more than 90% of power ratio is less than (-14 dB). Indeed, the existence of the power cost $-\pi^2_{jk}\alpha_{jk}$ in the utility function (15), diminishes the selfishness of eNodeBs that are tempted to transmit at full power on all RBs.

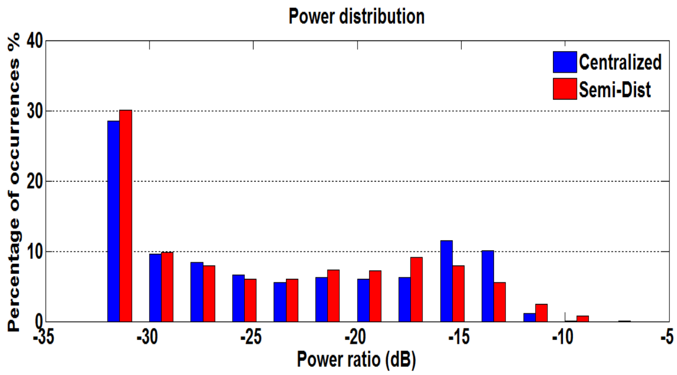

Fig 2. Percentage of power ratio distribution occurences for centralized vs. semi-distributed algorithms

Moreover, we can see that more than 30% of the power in the semi-distributed scenarios is around the minimum power level p_j^{min}. The highest SINR occurrences are obtained for power ratio levels ranging between -30 and -27 dB which is illustrated in Fig. 3 and 4.

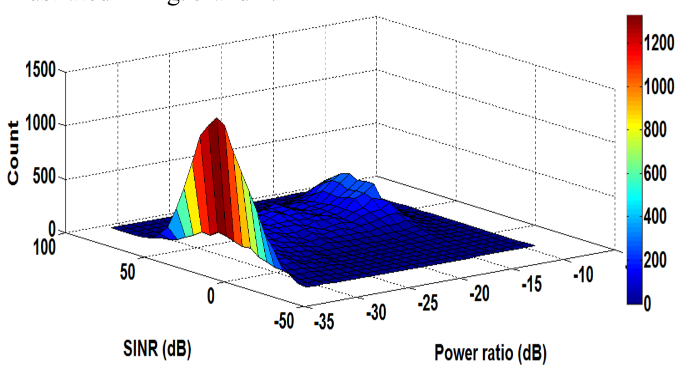

Fig 3. Occurences of SINR as function of power ratio for centralized algorithm

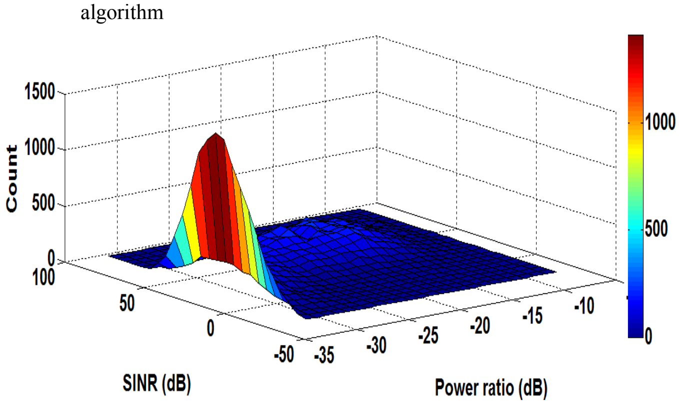

Fig 4. Occurences of SINR as function of power ratio for semi-distributed algorithm

We can see that the occurrences count of high SINR values is high for power level interval ranging between -30 and -27dB and -18 and -13 dB for the centralized approach. However, the same SINR occurrences' values are concentrated only on the power interval ranging between -30 and -27 dB for the semi-distributed approach.

In Fig. 5, we can see again the minor difference in SINR performances and power distribution between both approaches. Furthermore, the mean value of SINR, ranging between 30 and 40 dB, is obtained in the centralized approach for an average power value smaller than that of the semi-distributed scenario. Still, both power control schemes permit a considerable power economy in comparison with the Max Power policy, that uses full power P^{j}_{maz} for each eNodeB, as we can see in figure 6 where the power economy percentage for all eNodeBs vary from 53 to 77 % in comparison with the Max power policy.

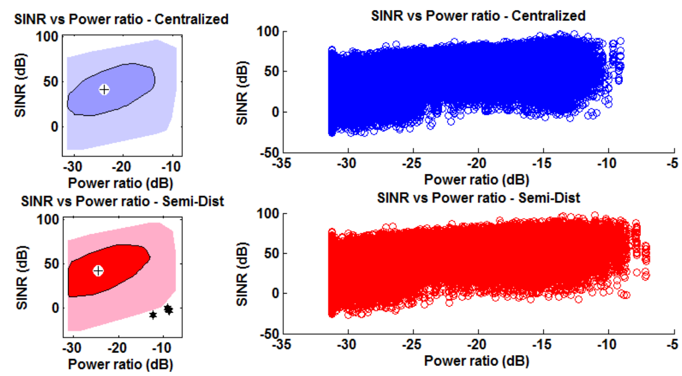

Fig 5. SINR and power ratio as a function of pathloss for centralized vs. semi distributed algorithms

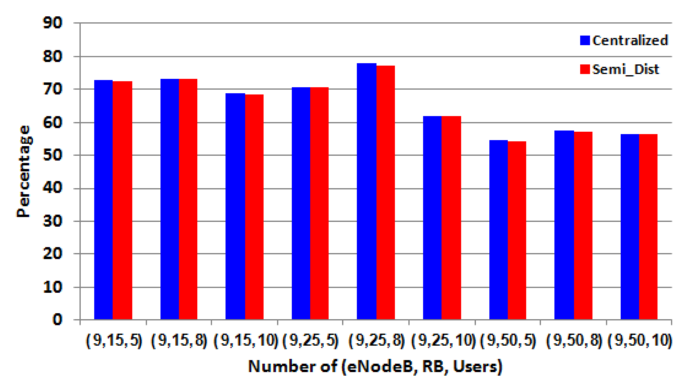

Fig 6. Percentage of power economy as a function of the number of eNodeB, RB and users for centralized vs semi-distributed algorithms

We can see the similarity of power economy efficiency between the centralized algorithm and the semi-distributed algorithm. This power economy is obtained while maintaining good performances as we can see in Fig. 7 where the utility function in (9a) is depicted as a function of the number of eNodeBs, RBs and users for the centralized, the semi-distributed and Max Power algorithms.

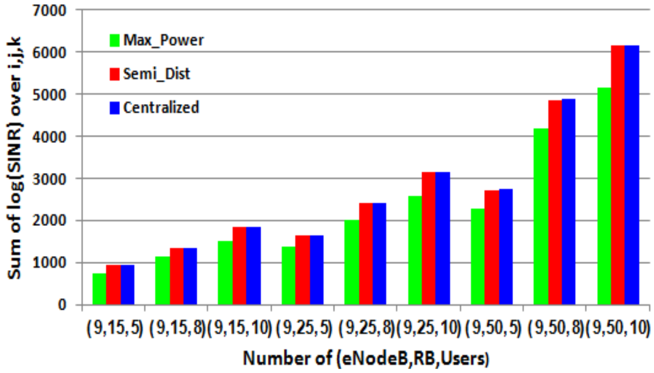

Fig 7. The Sum of log(SINR) as function of the number of eNodeB, RB and users for centralized, semi-distributed vs Max power algorithms

B. *Convergence Time*

In figure 8, we report the mean convergence time per eNodeB of the semi-distributed algorithm for various scenarios. We note that each eNodeB attains the NE within 32 to 87 iterations. At each iteration, one eNodeB is randomly selected to maximize its payoff function given in (15). The iteration period is equal one TTI (Transmit Time Interval), which equals 1ms in LTE.

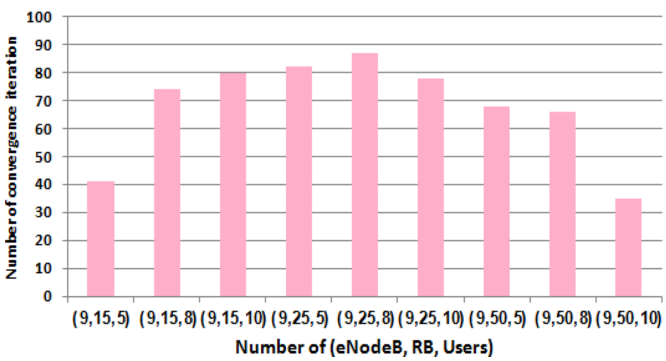

Fig 8. Total convergence time by eNodeB as function of the number of eNodeB, RB and users for semi-distributed algorithm

We noted during the extensive simulations conducted, that the power levels attain 90% of the values reached at convergence in less than 25 iterations. We can see that in Fig.9, where we represented the power distribution of 25 RBs for an eNodeB selected randomly and for which convergence time was equal to 87 iterations.

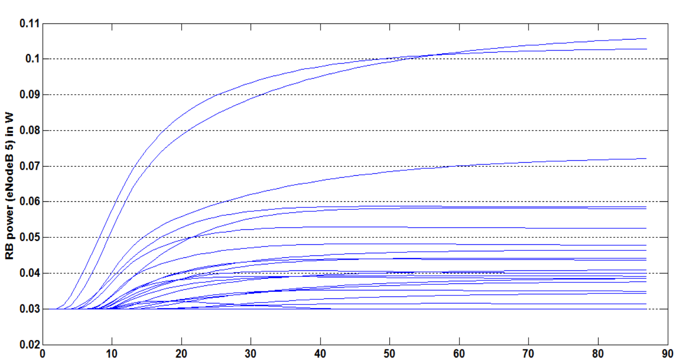

Fig 9. Power distribution by RBs before reaching convergence for semi-distributed algorithm

Low convergence time in conjunction with high performances is an undeniable asset for our semi-distributed schemes.

This result is corroborated in Fig.10 where we show that the utility function attains nearly its optimal value at 25 iterations. Hence, the fast convergence time, the near optimal results and the lower complexity degree of the semi-distributed approach makes it a very attractive solution.

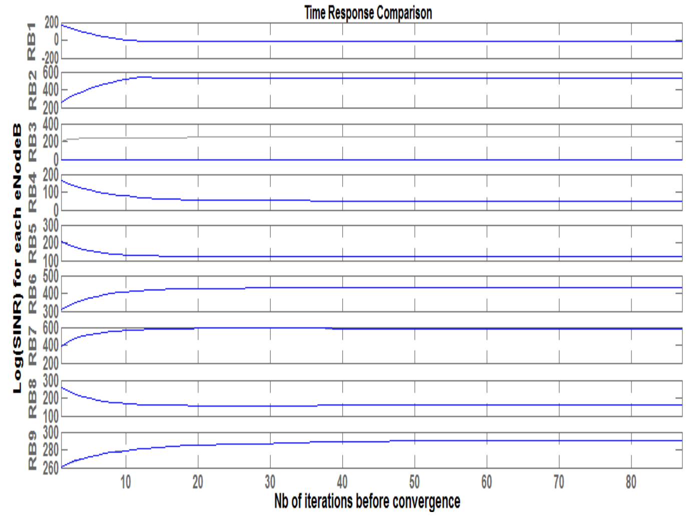

Fig 10. log(SINR) distribution by eNodeB before reaching convergence for semi-distributed algorithm

VI. Conclusion

In this paper, a joint scheduling and power control scheme is proposed as part of the LTE Inter cell Interference coordination process. The original problem is decoupled into a scheduling scheme and a power control scheme. We showed that, for the scheduling problem, proportional fairness has led to temporal fairness. As for the power control problem, a non-cooperative game resulted in a semi-distributed algorithm that astutely and efficiently set the power levels with relatively low convergence time. Numerical simulations assessed the good performances of the proposed approach in comparison with the optimal centralized approach.

VII. Appendix

A. *Proof of proposition 3.1*

In problem (7), constraints (7b) give $\sum_{k\in K}\sum_{i\in I(j)}\theta_{ik} \le K$ and constraints (7c) give $\sum_{i\in I(j)}\sum_{k\in K}\theta_{ik} \le |I(j)|$. Further, the objective function in (7a) can be written as:

$$\sum_{i\in I(j)}\sum_{k\in K}\log(\theta_{ik}). \tag{25}$$

Hence, we define a new scheduling problem less constrained than the initial one as follows:

$$\underset{\theta}{\text{maximise}} \quad U_j(\theta) = \log\left(\prod_{i\in I(j),k\in K}\theta_{ik}\right) \tag{26a}$$

$$\text{Subject to} \quad \sum_{i\in I(j)}\sum_{k\in K}\theta_{ik} \le \min(K, |I(j)|) \tag{26b}$$

$$0 \leq \theta_{ik} \leq 1, \forall i \in I(j), \forall k \in K. \qquad (26c)$$

As the objective function is non-decreasing, the optimal point must lie on equality constraint in (26b). Consequently, the sum of the θ_{ik} variables is constant and given by $\sum_{i\in I(j)} \sum_{k\in K} \theta_{ik} = \min(K, |I(j)|)$. Hence, the product of these variables is maximized when they are the same, i.e. for:

$$\theta_{ik} = \frac{\min(K, |I(j)|)}{K.|I(j)|}, \forall i \in I, \forall j \in J$$

This solution obeys the constraints of the original scheduling problem (7) but that it might not be an optimal solution for the latter. Let us suppose that $|I(j)| \geq |K|$ and θ^* is a solution vector for problem $(\widehat{P}_1)$ given by $\forall i \in I(j), \forall k \in K, \theta^*_{ik} = \frac{1}{|I(j)|}$. θ^* is a feasible solution for problem $(\widehat{P}_1)$ as it satisfies the constraints (7b) and (7c). Particularly, (7b) becomes an equality and (7b) is satisfied because $\sum_{k\in K} \theta_{ik} = \frac{|K|}{|I(j)|} \leq 1$. Let us demonstrate by contradiction that θ^* is an optimal solution for problem $(\widehat{P}_1)$. For any other solution of problem $(\widehat{P}_1)$, suppose that $\exists i' \in I(j), \theta_{i'k} = \frac{1}{|I(j)|} + \epsilon$. Then, to satisfy the constraints (7b), we should have $\exists i'' \in I(j), \theta_{i''k} = \frac{1}{|I(j)|} - \epsilon$. The objective of such solution is lower that of θ^* and the optimality of θ^* is proved.

B. Proof of proposition 3.2

We will prove that the resulting optimization problem (10) $P(\widehat{\pi})$ is convex; the first term of the objective is a linear function, thus concave (and convex). The second term contains log-sum-exp expressions which are convex. The opposite of the sum of convex functions being concave, this completes the proof of the concavity of the objective function. As for the new constraints: constraints (10b) are convex by virtue of the properties of the log-sum-exp functions and (10c) are linear functions and hence convex.

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